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Continuous river flow simulation: methods, applications and uncertainties

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Edited by Ian Littlewood

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Continuous river flow simulation: methods, applications and uncertainties

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Centre for Ecology and Hydrology
Wallingford

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Contents

Contents i

Preface iii

Unit hydrograph characterisation of continuous quick and slow response components of river flow: applications and uncertainties 1

Ian Littlewood

Response function techniques applied to drain discharge analysis 11

Adrian Armstrong and Steve Rose

Long-term low flow forecasting for French rivers by continuous rainfall-runoff modelling 21

Charles Perrin, Claude Michel and Vazken Andréassian

River flow simulation within ungauged catchments using a daily rainfall-runoff model 31

Andy Young

Classifying components of river basins for hydrological model parameter estimation 39

Anne Reungoat and William Sloan

On the evaluation of conceptual rainfall-runoff models using multiple objectives and dynamic identifiability analysis 45

Thorsten Wagener and Howard Wheater

Flood frequency estimation by continuous simulation (with uncertainty) 53

David Cameron, Keith Beven, Jonathan Tawn and Pamela Naden

Generation of spatially consistent rainfall data 59

Richard Chandler, H.S. Wheater, V.S. Isham, C. Onof, S. Bate, P.J. Northrop, D.R. Cox and D. Koutsoyiannis

Continuous simulation as a basis for national flood frequency estimation 67

Robert Lamb and Ann Calver

Affiliations of authors 77

Preface

This Occasional Paper is based on presentations at a National Meeting of the British Hydrological Society at the Centre for Ecology and Hydrology, Wallingford, 5th July 2001. More than 85 delegates received draft papers at the start of the meeting. These papers were subsequently reviewed and the authors invited to make revisions. The final papers appear here in the order that presentations were given at the meeting. Thanks are due to Howard Wheater (Imperial College, London; President BHS, 1999-2001), Keith Beven (University of Lancaster) and Bob Moore (CEH, Wallingford) for leading parts of the meeting, and for their review comments. A report of points discussed at the National Meeting is available in the BHS quarterly newsletter *Circulation* (volume 71, November 2001, pages 8-11).

Models that simulate the continuous (e.g. hourly, daily) response of river flows to storm precipitation and other environmental driving variables have many uses in hydrological science and environmental management. The topic is too wide to be covered in a single BHS Occasional Paper. However, the nine papers presented here deal with a range of modelling issues of interest to those concerned with characterising river flow regimes and advising environmental managers through the delivery of special studies, computer models, flood risk evaluation procedures, etc.

The floods of Easter 1998 and autumn 2000 reinvigorated interest in procedures for systematic flood risk estimation in the United Kingdom. The Institution of Civil Engineers report *Learning to live with rivers*[1] holds the view that we should undertake "… continual appraisal of the techniques we use, rather than just reassess our techniques following major flooding events – as has happened in the past." It is not sufficient simply to reassess and modify existing techniques. There is a duty to consider new techniques brought about by advances in computing power and modelling algorithms. Most of the papers in BHS Occasional Paper No. 13 provide an introduction to work that has been taking a forward look towards better systematic approaches to continuous rainfall-runoff modelling for flood and water resources hydrology. Other papers presented in this volume deal with continuous flow simulation in the contexts of low river flows and field drainage networks respectively.

Affiliations of all the authors are given on page 77.

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January 2002

[1] Institution of Civil Engineers (2001). *Learning to live with rivers*. Final report of the Presidential Commission to review the technical aspects of flood risk management in England and Wales. Institution of Civil Engineers, London, 84pp.

Unit hydrograph characterisation of continuous quick and slow response components of river flow: applications and uncertainties

Ian Littlewood

INTRODUCTION

Unit hydrograph theory has been a keystone of systematic applied catchment-scale hydrology in the UK since the Flood Studies Report (FSR) methods were introduced in the 1970s (NERC, 1975). More recently, the FSR unit hydrograph methodology was restated, essentially unchanged, in the Flood Estimation Handbook (IH, 1999). While unit hydrograph (UH) theory has stood the test of time, advances in computing power and applied mathematics have enabled new, more powerful, UH identification techniques to evolve and be applied. For example, the IHACRES methodology (Jakeman *et al*., 1990; Littlewood and Jakeman, 1994) allows UHs to be identified for dominant quick and slow flow components of streamflow, rather than for a single, fairly arbitrarily defined, 'FSR direct flow' component (involving prior baseflow separation).

Compared to the FSR event-based methodology, IHACRES identifies a UH for total streamflow (the sum of the quick and slow UHs) from continuous time series of rainfall and streamflow. Furthermore, continuous simulation of total streamflow (and the quick and slow flow components thereof) is enabled from which a Slow Flow Index (SFI), analogous to the well-known Base Flow Index (BFI), can be derived. Given a suitable 'loss module', the range of potential applications of rainfall-streamflow models based on classic unit hydrograph theory has been extended from its almost exclusive use historically for studies of floods to now include catchment characterisation over a wider range of flows.

With example applications of the PC-IHACRES v1.02 software package (Littlewood *et al*., 1997) the paper outlines major features of the underlying IHACRES methodology and presents some of the uncertainty issues commonly encountered when calibrating and using such models.

AN ABRIDGED INTRODUCTION TO IHACRES

IHACRES quick (q) and slow (s) unit hydrographs are of the same, parametrically constrained, form shown in Fig. 1. Input of unit effective rainfall, u_k, over a single data timestep (Δ), with $u_k = 0$ over all other time steps, generates the flow response shown, which has an exponential decay. In Fig. 1, the measurement units are assumed to be the same for effective rainfall and flow, e.g. millimetres.

Adopting the definitions of a and b given in Fig. 1 where, for illustration, they are set to 0.91 and 0.78 respectively, streamflow Q_k is modelled with a transfer

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function of the form given by equation (1). The quick and slow flow hydrographs are modelled by the first and second terms respectively in the bracketed part of equation (1), each term acting on effective rainfall. The mathematical background and development history of IHACRES are given by Jakeman *et al.* (1990).

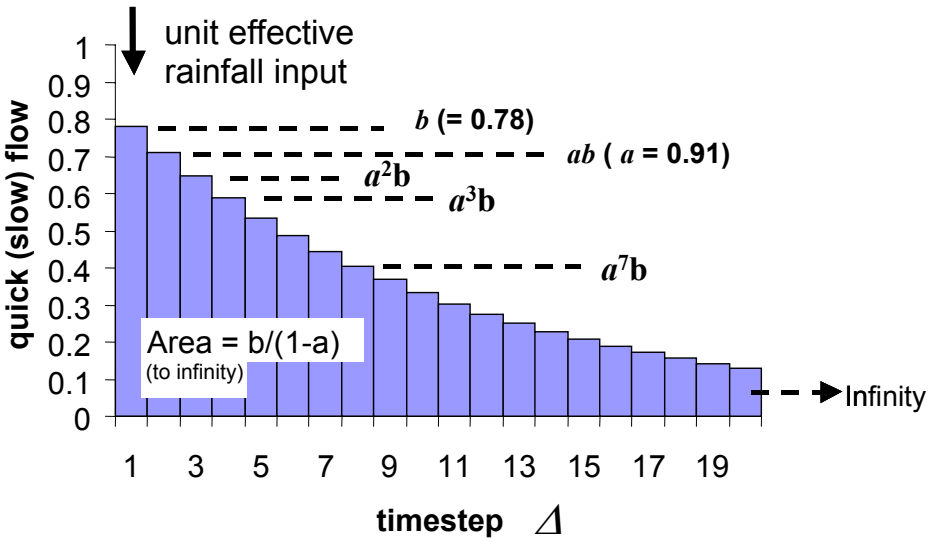


Figure 1. Form of the IHACRES quick (and slow) response unit hydrograph

$$Q_k = \left(\frac{b^{(q)}}{1 - a^{(q)} z^{-1}} + \frac{b^{(s)}}{1 - a^{(s)} z^{-1}} \right) u_{k-\delta} \qquad (1)$$

where δ is a pure time delay (an integer number of data timesteps in PC-IHACRES v1.02) and z^{-1} is the backward shift operator such that $z^{-1}x_k = x_{k-1}$

$$u_k = r_k \frac{(s_k + s_{k-1})}{2} \qquad (2)$$

$$s_k = Cr_k + \left(1 - \frac{1}{\tau_w(t_k)}\right) s_{k-1} \qquad (3)$$

$$\tau_w(t_k) = \tau_w \exp(0.062 f(R - t_k)) \qquad (4)$$

where: s_k is a catchment wetness index ($0 < s_k < 1$) (dimensionless); τ_w is a catchment drying time constant (e.g. days) given by the value of $\tau_w(t_k)$ at a reference temperature, R (°C); f is a temperature modulation factor (°C^{-1}); and C is a volume-forcing constant (mm^{-1}).

Assuming there are no significant groundwater exchanges with neighbouring, topographically defined, catchments effective rainfall, u_k, is the part of rainfall, r_k, that eventually leaves the catchment as streamflow. In PC-IHACRES v1.02, u_k is calculated by equations (2) to (4) but other 'loss modules' have been used within an IHACRES framework (e.g. Evans and Jakeman, 1998).

Leaving aside the pure time delay, δ, the model has six parameters: C, τ_w and f, plus any three of $a^{(q)}$, $b^{(q)}$, $a^{(s)}$ and $b^{(s)}$ (a simple function of the four UH parameters

is constrained to have a value of unity, so knowledge of any three defines the fourth). The value for C in equation (3) is calculated automatically by PC-IHACRES v1.02 such that, over a model calibration period, the sum of effective rainfall (Σu_k) is the same as the sum of observed streamflow. Calibration periods are selected to start and finish at times of low flows, preferably at the end of long hydrograph recessions when the storage of water in the catchment is at (or close to) its lowest level. It is then reasonable to assume that the difference in volumes of water stored in the catchment between the start and end of the calibration period is small and, therefore, that the volume-forcing imposed when C is calculated (as described above) is reasonable hydrologically. Conceptually, $1/C$ can be considered to be the capacity of a catchment wetness storage, i.e. when $s_k = 1$ the depth of water in this storage is $1/C$ mm (Post *et al*., 1998).

Parameters τ_w and f are selected (as a pair) by trial and error assisted by repetitive calibration of the UH parameters automatically by a 'simple refined instrumental variable' method (SRIV) (Jakeman *et al*., 1990). In the first instance, a 'best' model is selected on the basis of a trade-off between a high coefficient of determination (D_c) and a high average precision on the UH parameters (low %ARPE). Equations for D_c and ARPE are given in an Appendix to this paper. A pure time delay (δ) of an integer number of data timesteps is allowed in PC-IHACRES v1.02. This model parameter is selected simply by trial-and-error through repetitive operation of the software. For UK catchments and using daily data δ is typically either 0 or 1 day. Characteristic decay times (τ), average relative volumetric contributions to modelled streamflow (ν), and relative magnitudes of the quick and slow flow UH peaks (P) can be calculated from the a and b transfer function parameters in equation (1), using equations (5) to (10) in Table 1. Parameter $\nu^{(s)}$ is the Slow Flow Index (SFI) analogous to the well-known Base Flow Index, BFI. Parameters τ_w, f and C, and $\tau^{(q)}$, $\tau^{(s)}$, $\nu^{(q)}$, $\nu^{(s)}$, $P^{(q)}$ and $P^{(s)}$, are dynamic response characteristics (DRCs).

Table 1. Calculating DRCs from UH model parameters

DRC	Quick flow	Slow flow
Characteristic decay response times (Δ = data time step, e.g. 1 day)	$\tau^{(q)} = \frac{-\Delta}{\ln(a_1^{(q)})}$ (5)	$\tau^{(s)} = \frac{-\Delta}{\ln(a_1^{(s)})}$ (6)
Relative volumetric throughflow where $V = \frac{b_0^{(q)}}{1-a_1^{(q)}} + \frac{b_0^{(s)}}{1-a_1^{(s)}}$	$\nu^{(q)} = \left(\frac{b_0^{(q)}}{1-a_1^{(q)}}\right)\left(\frac{1}{V}\right)$ (7)	$\nu^{(s)} = \left(\frac{b_0^{(s)}}{1-a_1^{(s)}}\right)\left(\frac{1}{V}\right)$ ($\nu^{(s)}$ = SFI) (8)
Relative magnitudes of sub-unit hydrograph peaks	$P^{(q)} = \frac{b_0^{(q)}}{b_0^{(q)} + b_0^{(s)}}$ (9)	$P^{(s)} = \frac{b_0^{(s)}}{b_0^{(q)} + b_0^{(s)}}$ (10)

COMPARISON OF STREAMFLOW REGIMES

Table 2 gives (a) basic hydrological details of two very different catchments, the Teifi at Glan Teifi, Wales and the Eleshnitza at Vaksevo, Bulgaria, (b) their loss module DRCs and (c) goodness of model-fit statistics D_c and %ARPE. The relatively high-

Table 2. Catchment and model details

		Teifi at Glan Teifi	Eleshnitza at Vaksevo
(a)	Location	Wales	Bulgaria
	Area (km^2)	894	315
	Precip. (mm/yr)	1300	700
	Runoff (mm/yr)	960	214
	Mean runoff (%)	74	31
(b)	Model period	26 Jul 1982-29 Jul 1985	30 Sep 1986- 30 Sep 1988
	$f\,(^oC^{-1})$ $[R=20^oC]$	1.1	3.0
	τ_w (days)	10	4
	δ (days)	1	0
	$1/C$ (mm)	169	181
(c)	D_c	0.882	0.742
	%ARPE	0.03	0.71

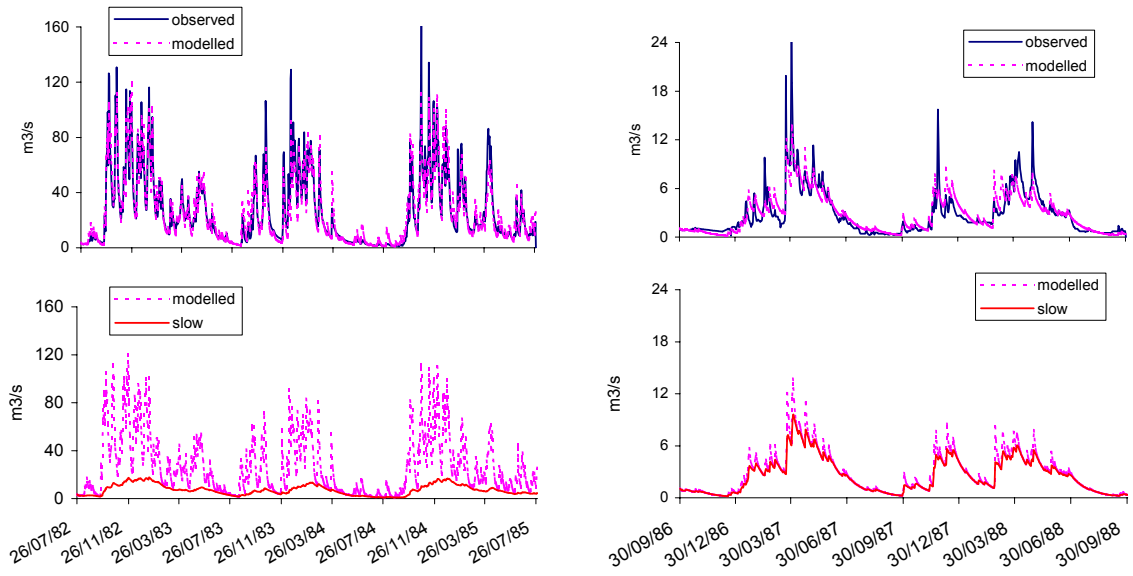
Table 3. UH module DRCs (with uncertainties)

DRC	Catchment	Quick flow $^{(q)}$	Slow flow $^{(s)}$
$\tau^{(.)}$ *(days)*	Teifi	2.97 (-7.4%,+8.4%)	39.9 (-23%,+43%)
	Eleshnitza	2.11 (-36%,+76%)	29.8 (-9.4%,+21%)
$v^{(.)}$	Teifi	0.731 (-6.1%,+5.7%)	0.269 (-16%,+17%)
	Eleshnitza	0.104 (-34%,+68%)	0.896 (-9.0%,+3.9%)
$P^{(.)}$	Teifi	0.969 (-1.5%,+1.0%)	0.031 (-35.5%,45.2%)
	Eleshnitza	0.570 (-15%,+15%)	0.430 (-21%,+19%)

rainfall Teifi catchment is nearly three times larger than the Eleshnitza catchment, which experiences much higher potential evaporation than the Teifi. Mean percentage runoff from the Eleshnitza catchment is very much less than for the Teifi. Table 3 gives the UH module DRCs for the two catchments, and Figs. 2a and 2b show the model-fits and hydrograph separations. The SRIV algorithm employed to estimate the UH parameters also yields indicative uncertainties on the *a* and *b* parameters in equation (1). Corresponding uncertainties in the unit hydrograph DRCs (at the 95% confidence level) are given in parentheses in Table 3.

The SFI ($v^{(s)}$) for the Teifi is 0.27, whereas it is about 0.90 for the Eleshnitza. Over the same respective modelling periods the BFIs are 0.56 and 0.72 (Table 2) . The characteristic quick (and slow) flow decay time constants ($\tau^{(.)}$) are similar for the two catchments but the uncertainties associated with $\tau^{(q)}$ (and $\tau^{(s)}$) are very different. For the Teifi, $\tau^{(q)}$ is estimated with much better precision than $\tau^{(s)}$ but the situation is

reversed for the Eleshnitza (Littlewood, 2000). A similar pattern can be noted for $v^{(q)}$ (and $v^{(s)}$).



(a) (b)

Figure 2. Model-fits and hydrograph separations (a) Teifi, (b) Eleshnitza

REGIONALISATION

From the foregoing it is evident that IHACRES is a parametrically parsimonious conceptual model (PPCM) that yields DRCs for quick and slow flow components of streamflow. In principle, therefore, it is a good candidate for estimating continuous river flow at ungauged sites, via relationships developed for gauged catchments between model parameters (rather, the DRCs because they have relevant physical interpretations) and physical catchment descriptors (PCDs), e.g. catchment size, slopes, etc. There can, however, be large uncertainties in the results from such studies.

In a regionalisation study employing daily data for 60 catchments in England and Wales, without sub-division by region or application of any other catchment 'similarity' scheme, Sefton and Howarth (1998) used IHACRES to derive the relationships between DRCs and PCDs given by equations (11) to (13). Unfortunately, the authors do not give the statistical uncertainties associated with these regional models, perhaps because they were very large. Other regionalisation studies using IHACRES are those by Post *et al*. (1998) and Post and Jakeman (1999).

$$\tau^{(q)} = AREA^{\,0.13}\, STMFRC^{\,-0.40} \qquad (11)$$

$$\tau^{(s)} = 6.78LFGH6 + 1.2CLASSB + 0.47OTHER_H \qquad (12)$$

$$\mathrm{SFI} = 0.196 + 0.007GW \qquad (13)$$

where LFGH6, CLASSB, OTHERH and GW are fractions of the catchment covered by classified soil types or hydrogeological units (mineral soils/shallow groundwater, semi-permeable mineral soils/no groundwater, all other HOST classes and groundwater respectively)

MODEL CALIBRATION AND SUBSEQUENT PERFORMANCE

Figure 4 shows an 11-year record of daily rainfall and streamflow for the Teifi at Glan Teifi, indicating eight overlapping, approximately 3-year, model calibration periods and an approximately 1-year period (X in Fig. 4). Table 4 gives details of the models (model #1 was calibrated over period #1, and so on). Table 5 records the coefficient of

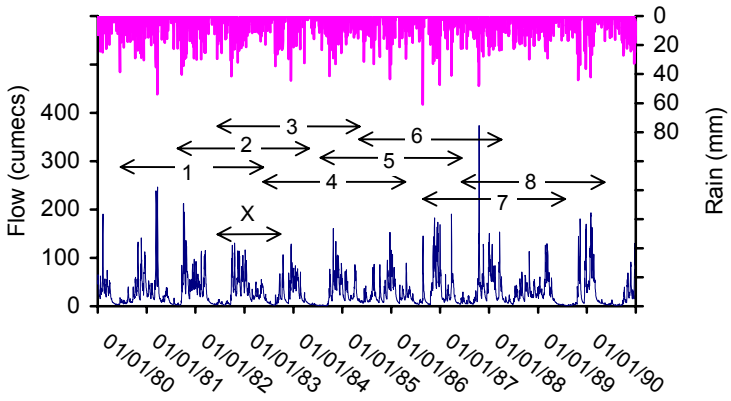


Figure 4. Teifi modelling periods

determination when each model was applied in simulation mode (D_s) over each non-calibration period. The bold typeface values in Table 5 correspond to the values of D_c in Table 4 (i.e. for model calibration periods, $D_s = D_c$).

Tables 4 and 5, which have been presented and discussed more fully elsewhere (Littlewood, 2001), reveal several interesting aspects of model calibration and subsequent performance. For example, it can be noted in Table 4 that although model #7 has a D_c–%ARPE pair similar to that for the other models it has an outlier value for $\tau^{(s)}$ of 395 days, compared to about 52 days for the other models. A model-fit plot (not shown) confirmed that model #7 performs very poorly at low flows.

Table 4. Teifi at Glan Teifi: calibration model-fit results

Period	TD	f ($^oC^{-1}$)	τ_w (days)	D_c	Bias (m^3/s)	% ARPE	$1/C$	$\tau^{(q)}$ (days)	$\tau^{(s)}$ (days)	SFI (υ_s)	BFI
#1	1	0.90	34	0.858	0.18	0.03	157.0	2.23	42.2	0.31	0.48
#2	1	1.00	34	0.866	-0.24	0.04	152.3	2.51	38.0	0.33	0.48
#3	1	1.00	37	0.886	0.00	0.03	163.1	2.98	47.0	0.27	0.50
#4	1	1.30	14	0.847	-0.33	0.04	63.2	2.45	39.7	0.45	0.51
#5	1	1.40	20	0.865	0.14	0.03	80.2	2.44	96.6	0.33	0.49
#6	1	1.30	24	0.871	0.47	0.03	94.7	1.88	63.2	0.33	0.48
#7	1	0.86	24	0.854	0.69	0.03	105.1	2.11	395	0.36	0.51
#8	1	1.30	17	0.864	1.53	0.04	69.4	1.81	38.8	0.43	0.56
mean$^{(1)}$	*1*	*1.13*	*25*	*0.864*	*0.30*	*0.03*	*110.1*	*2.30*	*95.1*	*0.35*	*0.50*
mean$^{(2)}$	-	-	-	-	-	-	-	-	*52.2*	-	-
#1 - 8	1	1.08	27	0.854	0.67	0.01	114.4	2.21	52.2	0.31	0.51
#X	1	0.75	33	0.928	0.01	0.05	168.0	2.83	50.0	0.29	0.51

mean$^{(1)}$: for models #1 to #8
mean$^{(2)}$: as mean$^{(1)}$ but excluding model #7

Although the D_c–%ARPE trade-off is superior to using only D_c as an objective function, it is fallible. The outlier $\tau^{(s)}$ for model #7 in Table 4 demonstrates that it is

crucially important when selecting a model as a characterisation of a catchment to assess whether or not its parameters (preferably its DRCs) are sensible hydrologically.

Table 5 Teifi at Glan Teifi: model performances in simulation mode (D_s)

Model	Period									Period	
	#1	#2	#3	#4	#5	#6	#7	#8	*mean*$^{(2)}$	#1-8	#X
#1	**0.858**	0.866	0.878	0.827	0.850	0.843	0.813	0.834	0.844	0.847	0.918
#2	0.855	**0.866**	0.884	0.831	0.845	0.829	0.799	0.822	0.838	0.840	0.924
#3	0.851	0.863	**0.886**	0.832	0.844	0.822	0.793	0.817	0.832	0.837	0.924
#4	0.824	0.841	0.870	**0.847**	0.852	0.842	0.818	0.846	0.842	0.841	0.895
#5	0.826	0.829	0.863	0.838	**0.865**	0.862	0.836	0.857	0.844	0.847	0.895
#6	0.834	0.830	0.853	0.822	0.857	**0.871**	0.843	0.864	0.843	0.850	0.887
#7	0.788	0.764	0.807	0.766	0.845	0.845	**0.854**	0.865	0.811	0.820	0.862
#8	0.828	0.833	0.858	0.835	0.855	0.864	0.839	**0.864**	0.844	0.849	0.883
mean$^{(1)}$	0.829	0.832	0.859	0.821	0.850	0.844	0.820	0.843	-	0.841	0.898
#1-8	0.849	0.851	0.871	0.834	0.861	0.863	0.835	0.858	0.853	**0.854**	0.906
#X	0.850	0.863	0.882	0.823	0.838	0.815	0.786	0.806	0.833	0.831	**0.928**

mean$^{(1)}$: mean D_s for models #1 to #8 (excluding the case where D_s=D_c, when relevant)
mean$^{(2)}$: mean D_s for periods #1 to #8 (excluding the case where D_s=D_c, when relevant)

Fig. 5 shows how the mean loss in model performance, in going from calibration mode to simulation mode (i.e. D_c minus *mean*^{2}(D_s) in Table 5), increases as D_c increases. This relationship is very strong (the outlier is for model #7). Models that perform very well over their calibration period, e.g. model #X with D_c = 0.928,

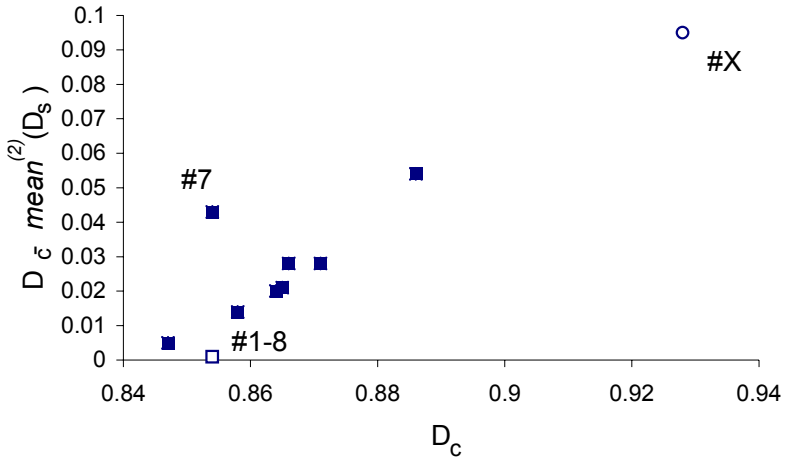


Figure 5. Loss in mean model simulation performance with increasing model calibration performance

tend to perform relatively less well in simulation mode on non-calibration periods than models with lower values of D_c. The deterioration in performance for model #X is so large that it is not as good a characterisation of the Teifi flow regime at Glan Teifi as most of the other models. In this sense, if the objective is to characterise the Teifi flow regime between 1980 and 1990, model #X is not a good model.

CONCLUDING REMARKS

The IHACRES methodology for identifying a UH for total streamflow from continuous rainfall and flow records has advantages (Littlewood, 1998) over earlier methods (IH, 1999) where a UH is estimated for a direct runoff component only using runoff event data. However, even when an IHACRES model has a high D_c (Teifi) the uncertainty associated with either the quick or slow characteristic decay time can be quite large (e.g. between -23% and +43% for Teifi $\tau^{(s)}$). Comparison of unit hydrographs for the Teifi and Eleshnitza has demonstrated that whether it is the quick or slow flow component that is least well identified is strongly influenced by their hydrological and climatological regimes .

With a loss module appropriate for the region and application in question, IHACRES is a strong candidate for the rainfall-runoff component of regionalisation schemes. However, the final selection of an IHACRES model (or any other PPCM) for each (gauged) catchment in such a scheme, from the set of models having similar goodness-of-fit statistics for the catchment in question, should be made with great care. In the case presented here for the Teifi it is evident that a good value of D_c can be a poor indicator of how well the model will perform in simulation mode, i.e. a model with a good D_c is not necessarily a good characterisation of the catchment. A two-parameter objective function comprising a trade-off between D_c and %ARPE is better than using D_c alone but it can still be unreliable (Teifi model #7). Additional criteria need to be inspected to ensure that the selected model is sensible hydrologically. Inspection of DRCs for outliers is an essential part of model selection. In combination with time series plots, comparison of flow duration curves for observed and modelled flows (not discussed here) is an efficient means of assessing model performance over a wide range of measured flows.

A model that performs very well in calibration mode over a short period of record (model #X) can be a poor characterisation of the catchment over a longer period and should, therefore, be treated cautiously. For a given catchment, parameter selection for a given model structure should include multiple split-sample testing of competing parameter sets and critical inspection of the rate at which model performance in simulation mode tends to decrease with increasing calibration mode performance (e.g. Table 5 and Fig. 5).

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Appendix

The coefficient of determination, D_c, is the proportion of initial variance in observed streamflow accounted for by the model. D_c is given by equation (A1) where σ denotes standard deviation, and ξ and y denote model residuals and observed streamflow respectively.

$$D_c = 1 - \frac{\left(\hat{\sigma}_{\xi}\right)^2}{\left(\hat{\sigma}_{y}\right)^2} \qquad \text{(A1)}$$

Jakeman et al., 1990 give a general equation for ARPE. Equation (1) can be written as a second-order transfer function of the form given by equation (A2). ARPE is then given by equation (A3).

$$Q_k = \left(\frac{b_0 + b_1 z^{-1}}{1 + a_1 z^{-1} + a_2 z^{-2}} \right) u_{k-\delta} \qquad \text{(A2)}$$

$$ARPE = \left[\left(\frac{\hat{\sigma}_{a_1}}{\hat{a}_1} \right)^2 + \left(\frac{\hat{\sigma}_{a_2}}{\hat{a}_2} \right)^2 + \left(\frac{\hat{\sigma}_{b_0}}{\hat{b}_0} \right)^2 + \left(\frac{\hat{\sigma}_{b_1}}{\hat{b}_1} \right)^2 \right] / 4 \qquad \text{(A3)}$$

Response function techniques applied to drain discharge analysis

Adrian Armstrong and Steve Rose

INTRODUCTION

Field drainage systems are particularly simple examples of discharge systems. They are normally small and discrete, with well defined boundaries. Normally they consist of simple patterns of pipes laid at regular spacings. During the field capacity period, rainfall causes a rise in the water table, and water then flows in a predictable way to the drains. Drainage theory is by now well developed to describe this process. However, some problems still remain unresolved, particularly with the analysis of drainflow data. This paper presents one attempt to understand drainflow behaviour using response function theory.

The particular problem we addressed was to characterise the decay of mole drainage systems. Mole drains are unlined drainage channels drawn in the soil at spacings of around 2m and depths around 0.5-0.6m below the surface (Spoor and Leeds-Harrison, 1999). The act of drawing the channels also introduces a critical system of cracks that provide an easy route for water to move from the surface. The channels themselves are discharged into gravel filled collector trenches containing permanent drainage pipe systems, normally installed at around 1m below the surface.

However, mole channels slowly decay over time. When new, mole drain channels offer highly effective drainage, but with time the cracks close up and the channels themselves decay. As a consequence, the drainage system becomes less effective, and eventually ceases to offer an acceptable degree of water table control. Normal agricultural practice is to renew mole drainage on a regular basis (recommendations for renewal are normally every 5 years). The first problem we address in this paper is the parameterisation of this decay.

EXPERIMENTAL DATA

ADAS has a large database of experimental observation drainflows through mole drainage systems. For this study, we used three major data sets, where high quality records of flow and rainfall have been recorded at half-hour intervals. The three sites (Table 1) involved are Brimstone Farm (Oxfordshire), North Wyke (Devon) and Cockle Park (Northumberland). At each of these sites, isolated plots have been established, and

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the drainflows recorded over several years from individual drain outfalls as one component in the study of drainage effects on nutrient and pollutant leaching. The problem was then to parameterise the response of the drainage system in such a way as to identify the performance of the system over a complete mole drainage cycle.

Table 1. Characteristics of the three experimental sites

	Brimstone Farm (Plot 9)	North Wyke	Cockle Park
Location (grid ref)	SU 248 947	SX 650 995	NZ 188 916
Soil series	Denchworth	Hallsworth	Dunkeswick
Subsoil clay %	62	40	35
Slope	2%	10%	3%
Main cropping	winter cereals	grass	cereals/grass
main drainage			
Spacing	46m	30m	40m
Depth	0.90 m	0.85 m	0.85 m
Secondary drainage	mole drains	mole drains	mole drains
Spacing	2.0 m	2.0 m	2.0 m
Depth	0.60 m	0.55 m	0.55 m
Moling dates	1993	1982 1986	1985

The characteristics of the discharge through these systems are illustrated by the example in Figure 1, which plots the rainfall and drain discharge for one plot from the North Wyke drainage experiment (Armstrong and Garwood, 1991). This shows that drainflow response to rainfall is very rapid, reaching peak discharge rates typically

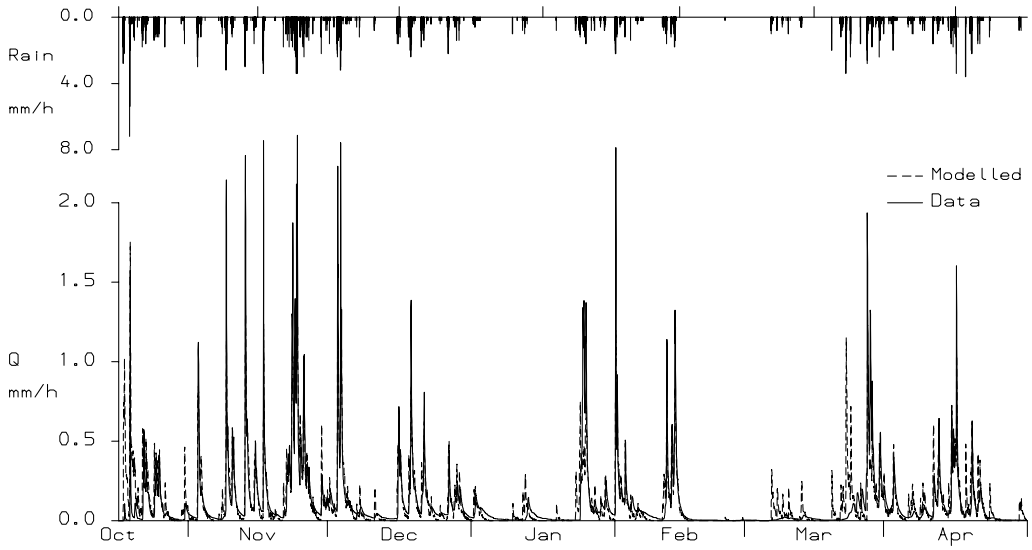


Figure 1. Example date set: rain and drainflow from North Wyke

between 1 and 2 mm/hour, with a rapid recession on the termination of rainfall. It is observed that for most of the winter, the drainflow record falls easily into a series of discrete discharge events, with periods of very low flow (or no flow) between. The speed of response is due to the very rapid travel times between the surface and the mole channels, through the extensive network of macropores in the soil (Hallard and Armstrong, 1992).

USE OF THE RESPONSE FUNCTION

We chose to use the response function as the primary tool for the characterisation of the hydrograph. Because it offers a description of the system response that is independent of the rainfall pattern, it affords a way of presenting the effects of the change in drainflow response that can be compared between years.

Response function theory derives from the general theory of linear systems (Dooge, 1973), and postulates a generalised relationship between the input and output of any system, of the form:

$$O(t) = \int_0^{t_m} K(\tau) I(t-\tau) d\tau \qquad (1)$$

in which the input, $I(t)$, is transformed to the output $O(t)$, by convolution with the kernel function $K(\tau)$. This kernel (the response function) can then be used describe the transformation of the input into the output.

Applying this analysis to drainflow hydrographs, we can immediately identify the inputs as precipitation P_t and the outputs as the discharges, Q_t. The kernel function then becomes an estimate of system behaviour independent of the nature of the rainfall inputs. Although a response function can be calculated from any paired set of inputs and outputs, it is most usually derived from equi-spaced series of data in time, so the discrete version becomes

$$O(t) = \sum_{k=1}^{\min(m,t)} I(k) H(t-k+1) \qquad (2)$$

in which the unit response function, $H(k)$ of length m, can be considered to be the response of the drainage system to a unit input. This is similar, but not identical to, the familiar instantaneous unit hydrograph. Generalised response functions, which have no constraints, are possibly more appropriate for some analyses, in that they allow for differing total flow volumes, and thus offer an empirical function as an objective generalisation of the rainfall-runoff response without any theoretical pre-suppositions.

In this study, we used the algorithm described by Bruen and Dooge (1984) to estimate the response function. However, in search of even greater generalisation, a simpler parameterisation of the response function was required. We observe that most response functions follow the classic unit hydrograph shape, with a rapid rise to a peak, and a gradual decline thereafter. We therefore attempted to generate parameters of the

response function, so reducing our overall analysis of the variation in system behaviour to a small number of parameters.

We thus split the response function into two sections, a rising limb and a falling limb, separated by a single peak. The parameters describing the rising limb of the hydrograph were relatively easy to define: we used the simple time to peak and peak height. Normally, these could be identified automatically with ease, by identifying the highest point in the response functions. Choice of the correct parameter to describe the hydrograph recession parameter was however less obvious. Rather than examine a wide variety of empirical recession curves, we took note of the results of Youngs (1985) which suggested a theoretical form for the recession of land drainage hydrographs. Youngs suggested that it is possible to predict the form of the recession using a transformation of time,

$$T = Cq_0^{\beta} t \tag{3}$$

where q_0 is the rate of flow at the peak, t is time, and C and β are constants. The discharge at any time, q_t is then given by (Youngs, 1985, eqn. 6)

$$\frac{q_t}{q_0} = \left(1 + T\right)^{-\frac{1}{\beta}} \tag{4}$$

These equations cannot easily be transformed into a regression style equation, but it has proved reasonably easy to estimate the parameters C and β by calculating the sum of squared errors and searching the parameter space for the minimum value. Adoption of this form reduces the recession component of the hydrograph to two parameters, which with the height and time of the peak reduces the whole response curve to just four parameters.

For the sequence of rainfall and runoff for the year 1984-85, Figure 1 shows, in addition to the observational data, the flows modelled from the rainfall using the Youngs

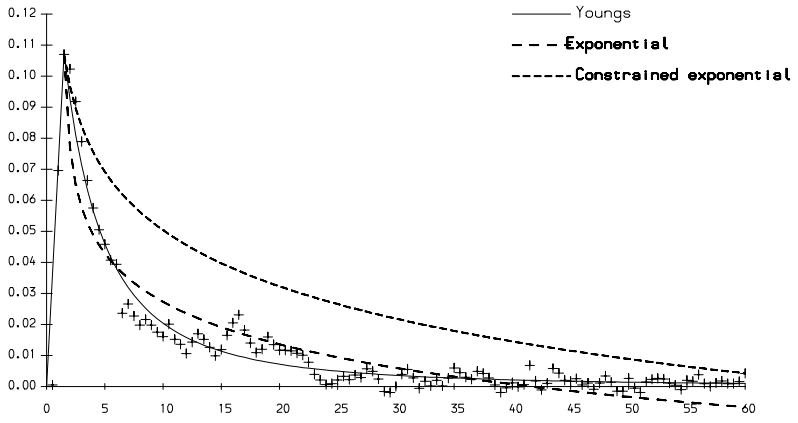


Figure 2. Response function derived from North Wyke, and fitted model response functions (the crosses mark the response function derived by the method of Bruen and Dooge (1984)

parameterisation of the response function. The fit to the data is good, and Figure 2 shows the response function with the Youngs parameterisation fitted. Other simple exponential regression forms (Armstrong, 2000) also shown in Figure 2 do not fit so well. The ability of the Youngs parameterisation both to model the data and to fit the observed response function is clearly shown by these results.

OBSERVATION OF MOLE DRAINAGE DECAY

The response functions were then used to examine the pattern of decay of drainage systems relying on mole drainage. For the three sites for which high quality data were available (Table 1) the response function was calculated and Youngs' form, given by equation (4), fitted to the recession. For each of the sites, the sequence of response functions is plotted in Figure 3.

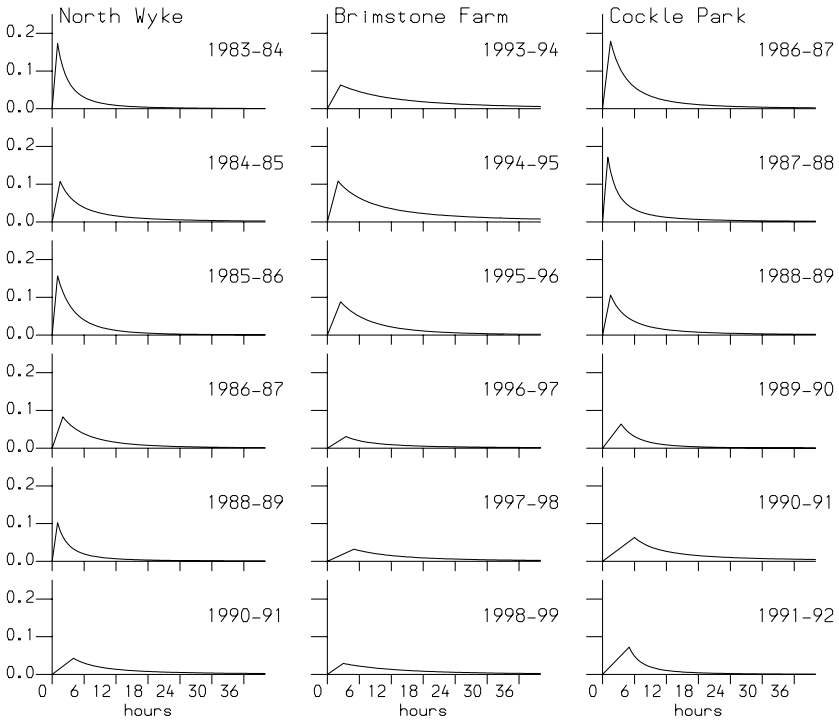


Figure 3. Response functions for successive years at North Wyke, Brimstone Farm and Cockle Park

These response functions all show a consistent pattern of behaviour, with the height of the hydrograph peak declining through the years after moling, and the recession becoming slower and more prolonged. This pattern is common to the sites even though the analyses cover different sequences of years, and so cannot be due to changes in rainfall patterns. Over the six-year period, the peak discharge rate falls from values of the order of 10-15% of the rainfall rate, to around 5% – a three-fold decrease. Simultaneously, the time to the response function peak also increases from around 0.5

hours to 3-4 hours. This is clearly an indication of much slower movement of water through the soil to the drainage system, and so indicates the hydrological effect of the decay of mole drainage systems.

At North Wyke, there is an observed increase in the hydrograph peak in 1986, which is a consequence of the renewal of the mole drainage in the previous year, suggesting that the sequence can be repeated with repeat moling.

INVERSE PARAMETER ESTIMATION

It is possible to take the analysis a stage further, and use it to derive estimates of the underlying soil parameters. This procedure of "inverse estimation" of parameters gives values that are related to the performance of the system, but also depends for its effectiveness on the correct identification of the relevant theory. The theory of Youngs (1985) which gave us the hydrograph shape also defines the parameters of the hydrograph shape from soil and drainage system variables. The two parameters, C and β can thus be defined. First, using a variable, α,

$$\alpha = 2(d/D)^{d/D} \tag{5}$$

where the drains are $2D$ apart, and the system is delimited by an impermeable barrier at distance d below the drains, these are given by

$$\beta = (\alpha - 1)/\alpha \tag{6}$$

and

$$C = (\alpha - 1)K^{1/\alpha}/SD \tag{7}$$

where K is the hydraulic conductivity of the soil, and S is the specific yield.

The problem we now address is to derive values of the unknown soil parameters from an observed sequence of drainage hydrographs. The system geometry factor α can be derived from knowledge of the drainage and soil system. If we then make an estimate of the specific yield, S, from standard soil properties, the only remaining unknown is the hydraulic conductivity, K. The specific yield is normally defined as the portion of total porosity that is filled between the two conditions of field capacity and saturation, which can be estimated from soil databases. This then gives us an expression for the remaining unknown, K, from the recession parameter, C.

$$K = \left[\frac{CDS}{(\alpha - 1)}\right]^{\alpha} \tag{8}$$

These ideas were explored using the same data from the North Wyke data set already shown in Figure 1. The mole drains are at 55 cm depth and 2 m apart, the half drain spacing, D, is 1 m and, as the depth of active soil below the drain is greater than

0.35 m, the limiting value for α of 1.36 is applicable. The specific yield (or drainable porosity) was estimated from the standard soil descriptions of the soil series, the Hallsworth soil series, using data derived from the SEISMIC database (Hallett *et al*.,1995), to be 7%.

The response function was then derived, fitted to the Youngs analysis, and the parameter *C* was fitted using a simple exploration of the response surface to give a least squares error solution. The results (Table 2) show low values, lower than those recorded using conventional measurement techniques such as the inverse auger hole test, which

Table 2. Inverse estimation of Ksat values from the North Wyke dataset

year	Ksat mm/d
83-4	0.226
84-5	0.138
86-7	0.182
87-8	0.120
88-9	0.226
90-91	0.103

generally give values between 1 and 10 mm/day. This analysis implies that the effective conductivity is at least an order of magnitude lower. However, these results are preliminary, and need to be confirmed by further analysis. Nevertheless they do show that the proposed methodology is capable of yielding solutions to the inverse estimation problem.

A further estimation of the parameters can also be attempted if additional data are available to give an independent observation of the drainable porosity. If this value could be estimated from data, then the use of library values in equation (8) could be replaced by a direct observation, and so an "effective" hydraulic conductivity estimate could be derived.

For many drainage experiments, there are, in addition to the discharge data, continuous records of the water table position. If the drainage discharge is envisaged as a simple response to the changes in water table position (as is required by classic drainage theory), we can then establish a simple water balance at the water table

$$M(t) = M(t-1) + (P(t) – PE(t) - Q(t)) / S \quad (9)$$

in which we see the change of water table height (M) as the effect of the additions of rainfall (P), loss from evapotranspiration (PE) and discharge through the drainage system (Q), all scaled by the drainable porosity parameter, S. This equation can then be re-arranged to give an estimate for the drainable porosity, which is the only unknown.

$$S = (P(t) – PE(t) - Q(t))/(M(t) - M(t-1)) \quad (10)$$

It is then possible to calculate the effective drainable porosity for every time increment, and perhaps to establish a time series for this variable. This has been attempted for a 25-day subset of the data set used from the North Wyke drainage experiment, and the results are shown in Figure 4. This diagram shows in turn, the rainfall, water table, discharge and the drainable porosity calculated from equation (10). It can be seen that the method does not yield a consistent estimate of the drainable porosity, but that the calculated value oscillates in sign as well as varying in value. However, the pattern is not totally random and, if clarified by heavily smoothing the output series, shows a consistent pattern of a rise in apparent porosity in the early stages

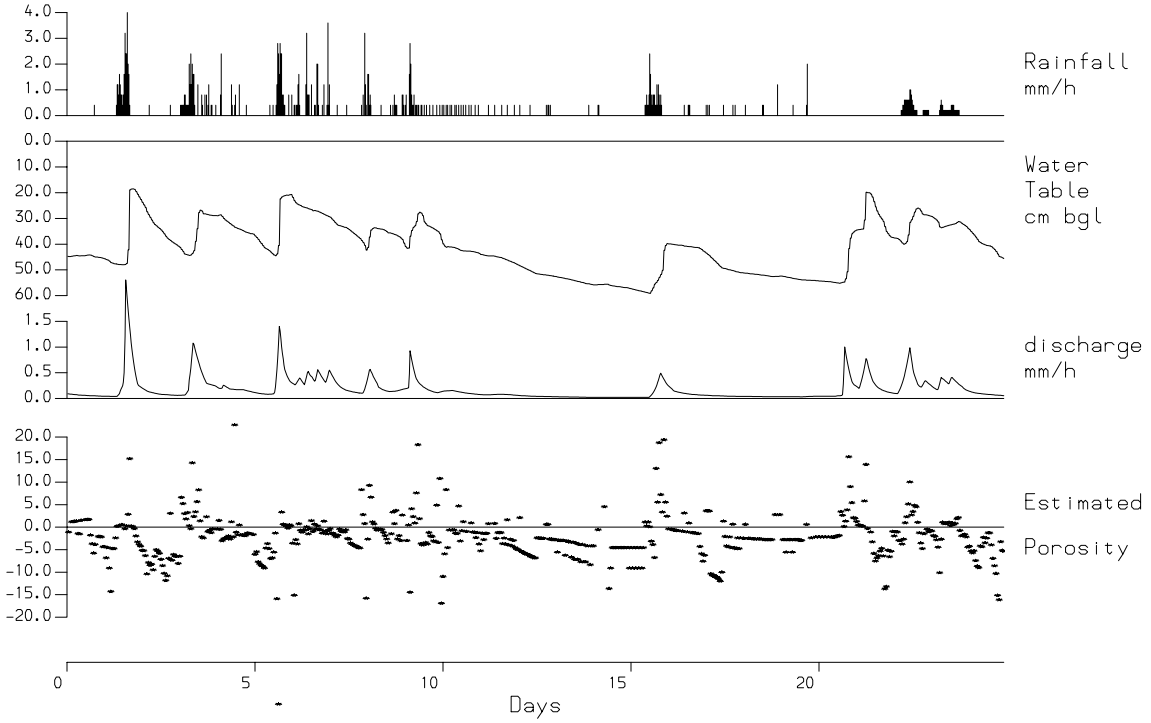


Figure 4. North Wyke, attempted inverse estimation of drainable porosity. The graph shows at half hourly increments, from top to bottom,: rainfall, water tables depth below ground level, discharge, and drainable porosity calculated from equation [10]

of every discharge event, and then a fall to negative values during the recession. These are the consequence of the fact that the water tables appear to lag behind the discharge peak, and then recede to their base level much more slowly.

There are two possible reasons for this mode of behaviour. The first is that the simple balance analysis of equation (9) does not take into account any lag times between inputs and outputs of the system. Hence the time of travel between soil surface and water table for incoming rainfall is ignored by the simple analysis. The second explanation could be that the effect is a consequence of the lag time of the instrumentation used (Hvorslev, 1951). The data presented are collected from recording meters inserted into open augur holes (Talman, 1983) and the lag time to transfer the volumes of water

involved into and out of the open augur hole is sufficient to create the effects observed: both the slower response to a rainfall event, and the longer recession. Unfortunately, neither the instrumentation nor this analysis can distinguish between these two effects, and it is indeed possible that the observations include the combination of the two. In these circumstances, the proposed analysis cannot be used to derive an inverse estimation of the drainable porosity. This result could not then be inserted into equation (7) to give a totally data-generated estimate of effective hydraulic conductivity.

CONCLUSION

These results have indicated the utility of response functions for the analysis of drain flows. In the first set of data, they yielded an objective generalisation that was capable of identifying the decline of mole drainage efficiency with time. In the second, preliminary analysis has demonstrated that it is possible use the response functions as a tool for deriving inverse estimation of system parameters. However, simple analyses using water table data to estimate drainable porosities, but which ignore travel times, fail to provide the additional information required to provide usable estimates of system performance.

ACKNOWLEDGEMENT

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Long-term low flow forecasting for French rivers by continuous rainfall-runoff modelling

Charles Perrin, Claude Michel and Vazken Andréassian

INTRODUCTION

Flood forecasting is a major concern of modern hydrology in an operational context, since extreme events can endanger human lives and cause great damage in urban areas, as shown by many examples in recent years throughout Europe. Comparatively, there has been less interest devoted to low flow forecasting, though such tools can prove very useful in water resources management systems. For example, low flow forecasts can help water stakeholders choose adequate reservoir operations to secure a satisfactory water supply to populations or to meet the requirements of agricultural activities during dry summer periods. Forecasts can also help to maintain good standards of in-stream water quality and quantity for ecological life (dilution of pollutants originating from human activities, output of waste-water treatment plants and diffuse agricultural pollution) and operational compliance with minimum flows required by law.

A deterministic short-term forecasting approach (applied typically for flood forecasting) is not very useful in the context of low flows since flow variations at that level are quite small. In contrast, it seems more relevant for water resources planning and management to employ medium or long-term forecasts (typically one or two months ahead). This can be done following a probabilistic approach. For example, a water manager will want to know, one or two months ahead, the probability of streamflow being below a threshold, given different probable rainfall scenarios.

To answer this question, a methodology is proposed using both a stochastic rainfall model and continuous rainfall-runoff modelling. In the following sections, the methodology is detailed. Both the rainfall generator and the rainfall-runoff model are then described. Lastly, the results of an application on two French basins are presented and discussed.

METHODOLOGY

One of the strengths of rainfall-runoff models originates from their detailed and continuous accounting of amounts of water that fill and empty catchment storages. They include therefore a memory of past moisture conditions that will play a key role on the determination of future flows in the more or less long-term.

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This characteristic makes models applicable for forecasting applications. Short-term forecasting is especially useful to anticipate flood events. Specific updating methodologies were developed to apply rainfall-runoff models in this context (see e.g. Yang and Michel, 2000). Such short-term approaches have sometimes been applied for low flows. For example, Campolo *et al.* (1999) used an artificial neural network to forecast low flows six days ahead on the Arno River in Italy. However very little interest was focused on long-term forecasting in those periods when flows are low and water management must be undertaken on a longer time scale.

The proposed methodology attempts to answer this issue. It uses a stochastic rainfall generator and a rainfall-runoff model and is based on the following steps:

1. Parameters of the rainfall-runoff model are first calibrated against available time-series of rainfall, potential evapotranspiration and streamflow. One must make sure that the ability of the model to characterise catchment hydrological behaviour and to simulate low flows on the catchment of interest is satisfactory according to appropriate criteria. At this stage, several models (or models versions) can be tested to select the best performing model on the study catchment. The parameters of the rainfall model are calibrated to characterise precipitation dynamics over the catchment.
2. One must choose the target period for which forecast flows are required and the day when the forecast is to be made. For example, a requirement might be to know on 1st June what will be the flow in the second half of July, i.e. for a period of 15 days after a lead time of 45 days. For the target period, a specific variable such as mean streamflow can be chosen.
3. The approach is based on a conditional forecast, which depends on rainfall scenarios. Starting from the day of forecast, a large number (N) of rainfall scenarios are generated and the model is run until the end of the target period. Then mean flow (or the variable of interest) can be computed. For each of the N scenarios, one starts from the same initial conditions. One can statistically analyse the sample of N results by building confidence intervals giving for example 10%, 50% and 90% percentiles that will characterise the distribution of results.

The methodology uses the precipitation model and the continuous rainfall-runoff model presented in the following sections.

STOCHASTIC RAINFALL MODEL

The model used to generate rainfall scenarios is based on a Markov chain procedure. As a lumped approach is used in this study, there is no need to model the spatial structure of rainfall. Only the temporal occurrence of rainfall will be modelled. The model distinguishes three precipitation states: (1) no rainfall, (2) moderate rainfall below a threshold defined as a fraction of mean rainfall (calculated for rainy days) and (3) rainfall exceeding the previously defined threshold.

As noticed by Smith (1993), two basic assumptions arise in modelling precipitation amounts associated with rainfall occurrences: selecting a distribution function and specifying the temporal correlation of precipitation amounts.

The distribution functions chosen to characterise states (2) and (3) are exponential and Weibull functions respectively. Here the rainfall state for a given day will be linked only to the state of the previous day. A daily time step is adopted in this study. To account for seasonal variability of rainfall occurrence, the rainfall simulation model is made dependent on each month of the year.

In each case, the parameters of both distribution functions were calibrated and the coefficients of the transition matrix between states were calculated using all the available rainfall information. When fitting distribution functions, care was taken to respect means to avoid any bias when simulating rainfall. Stochastic rainfall simulation was performed by using random number generators as detailed by Press *et al.* (1992).

GR4J RAINFALL-RUNOFF MODEL AND PARAMETER ESTIMATION

The proposed methodology can be applied with whatever rainfall-runoff model is available. For a specific application, it is obviously better to test different models and select the best performing one. In the context of this study, since emphasis is given to low flows, it must be checked that this range of streamflow is well simulated by the model.

As the proposed methodology is aimed at being applied in an operational context, a rainfall-runoff model has been chosen whose lumped and conceptual structure implies low data requirements. Here the methodology is applied to a modified version of the daily GR3J model proposed by Edijatno *et al.* (1999). This four-parameter version (GR4J) has been shown to be more robust and versatile than the GR3J model on a large sample of catchments (Perrin, 2000). It was therefore preferred to the GR3J model. Details of the model are given in Figs. 1 and 2.

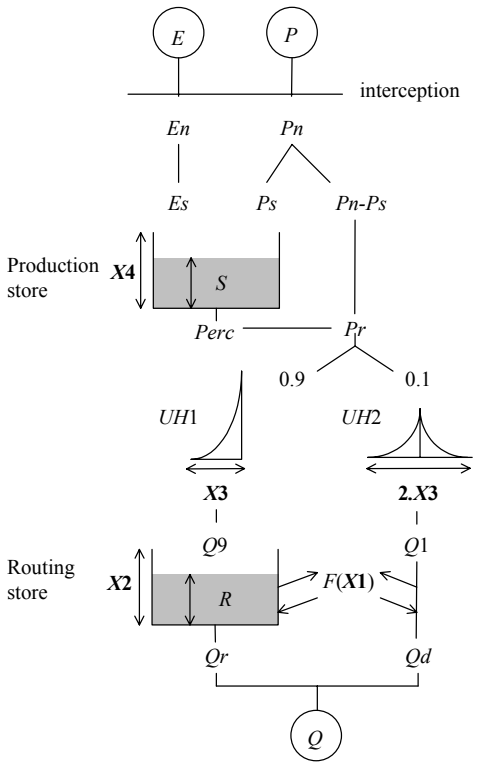


Figure 1. Schematic diagram of the modified 4-parameter version of the GR3J model

One of the original features of the model is its extreme parsimony, which helps to overcome the problem of equifinality. The GR4J model structure includes two water balance parameters (X1: water exchange coefficient; X4: capacity of production store) and two transfer parameters (X2: capacity of the non-linear routing store; X3: unit hydrograph time base).

Given its low number of parameters, the model can be calibrated with simple calibration techniques. Here model calibration was performed by a local optimization algorithm called the 'step-by-step' method. The principle of the method can be found in Edijatno *et al.* (1999). Starting from average parameter values, the procedure searches an optimum parameter set in the parameter space by maximising an objective function. The method proved efficient in the case of the GR4J model proposed here (see e.g. Nascimento, 1995; Perrin, 2000).

Interception (E : potential evapotranspiration; P : precipitation)

$$\text{if } P \geq E,\ Pn = P - E,\ En = 0$$
$$\text{else}\quad Pn = 0,\quad En = E - P$$

Production

$$Ps = \frac{Pn.\left(1 - \left(\frac{S}{X4}\right)^2\right)}{1 + Pn.\left(1 + \frac{S}{X4}\right)} \qquad Es = \frac{En.\frac{S}{X4}\left(2 - \frac{S}{X4}\right)}{1 + \frac{En}{X4}.\left(2 - \frac{S}{X4}\right)} \qquad S = S + Ps - Es$$

Percolation

$$Perc = S - \left(S^{-4} + \left(\frac{9}{4}.X4\right)^{-4}\right)^{-\frac{1}{4}} \qquad S = S - Perc \qquad Pr = Pn - Ps + Perc$$

Unit Hydrograph ordinates

$$UH1(j) = SH1(j) - SH1(j-1)$$

with $\quad 0 \leq j \leq X3, \quad SH1(j) = \left(\frac{j}{X3}\right)^{\frac{5}{2}}$

$\qquad j > X3, \quad SH1(j) = 1$

$$UH2(j) = SH2(j) - SH2(j-1)$$

with $\quad 0 \leq j \leq X3, \quad SH2(j) = \frac{1}{2}\left(\frac{j}{X3}\right)^{\frac{5}{2}}$

$\qquad X3 < j \leq 2.X3, \quad SH2(j) = 1 - \frac{1}{2}\left(2 - \frac{j}{X3}\right)^{\frac{5}{2}}$

$\qquad j > 2.X3, \quad SH2(j) = 1$

Water exchanges

$$F = X1.\left(\frac{R}{X2}\right)^{\frac{7}{2}}$$

Direct flow component

$$Qd = \max(0, Q1 + F)$$

Non linear routing store

$$R = \max(\varepsilon, R + Q9 + F) \qquad Qr = R - \left(R^{-4} + X2^{-4}\right)^{-\frac{1}{4}} \qquad R = R - Qr$$

Total streamflow

$$Q = Qr + Qd$$

Figure 2: Process equations in the modified 4-parameter version of the GR3J model

The choice of the objective function was made by considering the application of the hydrological model. Since low flows are of interest here, the selected objective function is a modified Nash and Sutcliffe (1970) criterion calculated by:

$$Cr1(\%) = 100.\left[1 - \frac{\sum_{i=1}^{n}\left(\sqrt{Q_{obs,i}} - \sqrt{Q_{calc,i}}\right)^2}{\sum_{i=1}^{n}\left(\sqrt{Q_{obs,i}} - \overline{\sqrt{Q_{obs}}}\right)^2}\right]$$

where $Q_{obs,i}$ and $Q_{calc,i}$ are observed and simulated streamflow values at time step i, and n is the total number of time steps in the test period. A prior square root transformation was applied to streamflow values to lower the importance of flood events and to give more importance to low flows.

In the following, two complementary criteria are also used to judge model performance: the same criterion as above, with a prior logarithmic transformation, which puts even more weight on low flows ($Cr2$), and the original Nash and Sutcliffe criterion calculated using untransformed streamflow values ($Cr0$).

ASSESSMENT OF THE METHOD

This method can be assessed on catchments where several years of data are available. It can be applied retrospectively for all the years of the available data record (say m years). The natural variability of mean flow over the target period can be characterised by either by making a continuous simulation over N years or by studying this variability in the available streamflow record if it is long enough. On this basis, the corresponding confidence interval (called hereafter a priori statistics) can be assessed in a similar way as previously.

By comparing the ranges of the confidence intervals calculated from the multiple run in the conditional forecasting procedure from one year to another, it is possible to assess the impact of initial conditions at the day of forecast on the flows over the target period. These ranges can also be compared with that of the a priori statistics. These comparisons will indicate if the long-term forecast is possible on the study catchment: in this case, the confidence intervals are significantly influenced by initial conditions.

CATCHMENTS AND DATA

Two French catchments were selected to illustrate how the methodology can be applied. Instead of testing several models on each catchment and selecting the best performing one, two French basins were deliberately chosen on which the GR4J model performs satisfactorily.

The first catchment is situated in the western part of France, in the Brittany region, with the gauging station located at Le Drennec on the Aber Ouest stream. The catchment is quite small (24 km²) and mainly agricultural. Daily time-series of rainfall and streamflow were available for the period 1972-1980 (1972 was used for model warm-up). Summer 1976 was exceptionally dry. Two raingauges were used to compute areal rainfall. Long-term mean Penman potential evapotranspiration (PE) data were available at a ten-day time step. Mean annual PE is 670 mm and mean annual rainfall is 1180 mm over the study period.

The second catchment is situated in the eastern part of France on the Moselle river. Catchment size to the gauging station at Custines is 6830 km². Data were available for the period 1979-1990 (1979 was used for model warm-up). A total of 21 raingauges spread over the catchment were used to compute mean catchment precipitation. Mean annual PE is 660 mm, and mean annual rainfall is 1030 mm over the study period.

Note that both catchments have quite a high base flow index (BFI) of 79% and 63% respectively. They have the same yield coefficient of 52% over the study periods.

RESULTS

The model was calibrated on each catchment over the whole period of record. Results (criteria and parameters values) are detailed in Table 1. The model performs quite

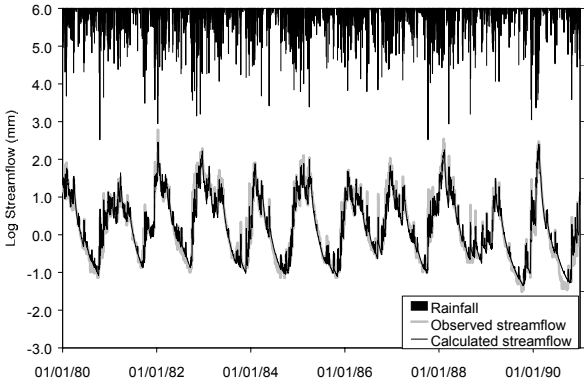
satisfactorily on both catchments. *Cr*0 and *Cr*2 values indicate that the model manages to fit flood peaks and low flows with good accuracy. The model seems therefore reliable for low flow simulation.

Table 1. Calibration results for the two study catchments

		Criteria			Parameters			
Catchment	Calibration period	*Cr*1 (%) (OF)	*Cr*0 (%)	*Cr*2 (%)	X1 (mm)	X2 (mm)	X3 (d)	X4 (mm)
Aber Ouest at Le Drennec	1980-1990	97.0	95.9	97.1	-0.04	788	1.40	255
Moselle at Custines	1973-1980	91.6	88.7	91.7	0.81	131	2.48	396

Values of the UH parameter (X3) indicate that the Aber Ouest catchment reacts more quickly than the Moselle catchment, which is partly due to its smaller size. Significant differences are observed in the capacities of model storages. A high X2 value (routing store) in the case of the Aber Ouest indicates that this catchment has a large storage capacity of water that can be made available for streamflow, which can be linked to the high BFI value observed for this catchment. However, the larger capacity of the production store in the case of the Moselle catchment shows that its soils may have a longer memory of antecedent moisture conditions.

(a)



(b)

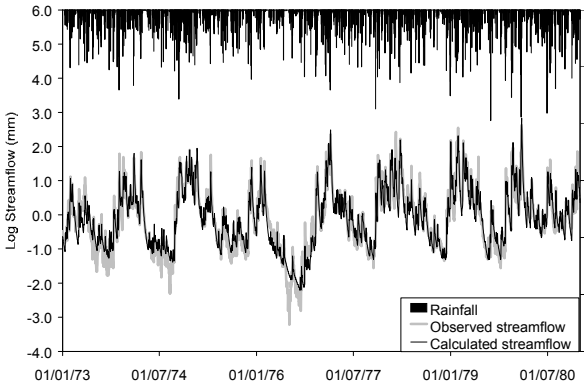


Figure 3. Observed and simulated streamflow time-series, (a) Aber Ouest and (b) Moselle

Fig. 3 shows the streamflow time-series obtained in calibration mode for both catchments. Logarithmic transformed streamflow values were used in these plots to better illustrate low flow simulation quality.

Parameters of the distribution functions of the stochastic rainfall simulation model were calibrated and coefficients of the rainfall generator calculated for both catchments on the whole sample of available data (for lack of space, results are not discussed here).

The extent of low flow periods (defined by an arbitrary threshold of one third of mean streamflow) is variable from one catchment to another. On the Aber Ouest basin, low flow periods generally start at the beginning of August to end in October with a minimum value in late September. On the Moselle catchment, the beginning of low flow periods is more variable (from June to August) but the first peak flows in Autumn often occur at the end of October.

Considering that the water demand is generally high in August, the second and third weeks of August were chosen as the target period mean flow to be forecasted. The day on which the forecast was made was 1st July, which means that an attempt was made to conditionally forecast streamflow almost 40 days ahead.

The methodology previously described was tested by applying it for all years of available data. A large number of scenarios was performed (N = 400). Results on the target period are shown in Fig. 4. For each catchment, the following information has been shown on the same graph:

- information related to the observed or simulated streamflow on the whole available record for the target period: for each year, the observed and simulated streamflow using actual observed rainfall are given. The 80% confidence interval of streamflow calculated on the available streamflow data is also provided (a priori statistics).
- information related to the forecasting approach, with 80% confidence intervals and median values for simulations using the rainfall generator.

For each catchment, simulated and observed streamflow are in good agreement, which indicates that the model is fairly accurate on the target period. In the case of the Aber Ouest catchment, it can be seen that the predicted 80% confidence intervals are quite narrow in comparison with the a priori statistics and that they significantly differ in range from each other. This means that the initial conditions on 1st June have a strong influence on flows for the second and third weeks of July and that the long-term forecast for that lead time and that target period is possible on this catchment.

For the Moselle catchment, the confidence intervals are both far less different from each other and their ranges are very similar to that of the a priori statistics. Although the model performs reasonably well on the target period, the initial conditions on 1st June do not seem to have a significant influence on flows 40 days later. Note, however, that in 1976 which was an exceptionally dry year, whatever the rainfall scenario over the 40 days, the flows would be low on the target period. Except in these exceptional conditions, the long-term forecast does not seem very informative for this catchment.

The difference between both catchments may be linked to the optimised parameter values. The routing store has a much larger capacity (X2) in the case of the Aber Ouest catchment, which means that its routing part has a longer memory than for

the Moselle catchment. This may explain the better ability of long-term forecasting on the Aber Ouest catchment.

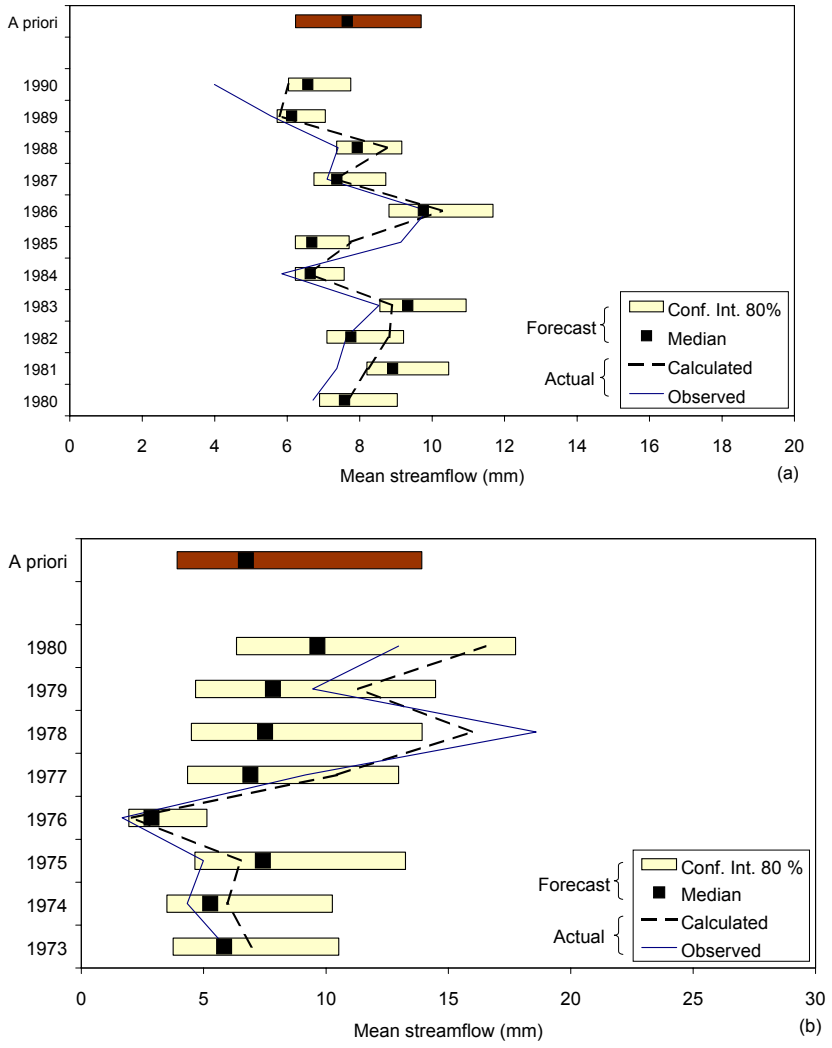


Figure 4. Application results of the long-term forecasting methodology for (a) the Aber Ouest and (b) the Moselle

CONCLUSIONS

This paper has presented a methodology to provide long-term forecasts in low flow conditions. Such forecasts can be very useful for water managers and practitioners. The method is based on the joint use of a stochastic rainfall model and a continuous rainfall-runoff model. It makes a conditional forecast of flow given a large number of rainfall scenarios.

The methodology was assessed on two catchments in France. For one of the catchments, it was shown that the methodology can be applied with good confidence. For the other catchment, the results indicate that although the rainfall-runoff model performs

satisfactorily in low flow conditions, the long-term forecast is unsuccessful. This is due to its relatively short memory length which does not enable such a long-term forecast.

Further research is needed to understand the link between parameter values of the hydrological model and the applicability of the forecasting method on a larger number of catchments.

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River flow simulation within ungauged catchments using a daily rainfall-runoff model

Andy Young

INTRODUCTION

Information on the magnitude and variability of flow regimes at the river reach scale is a central component of most aspects of water resource and water quality management. For some activities, such as the setting of discharge consents and licensing of small abstractions, it is sufficient to encapsulate this information using a statistical description of the flow regime. The flow duration curve (NERC, 1980) is an example of this type of analysis. However, there are many applications for which a time series of stream flows is required. These include the assessment of yield for water resource schemes, the in-stream flow requirements of aquatic flora and fauna and the assessment of the impacts of climate change at the catchment scale. Currently, the low-cost solution to the requirement for historical time series of river flows at ungauged catchments is to transpose gauged data from a similar, nearby, gauged catchment.

The overall objective of the research summarised in this paper was to develop a rainfall-runoff model for predicting the natural variation of daily stream flows within a catchment, without recourse to the calibration of model parameters against observed streamflow data. Implicit within this objective is a requirement that the model parameters can be estimated from the physical characteristics of the catchment. The focus of the research was on an operational requirement for a quick, consistent and reliable method for simulating historical streamflow records within ungauged catchments. Hence, as part of this overall objective, it was essential that the data required by the model are restricted to those data that are readily available in a digital form.

The temporal resolution for both managing and regulating water-use is commonly between a week and a month. However, for many water-use schemes, and cases where the impact of a scheme on aquatic flora and fauna is to be assessed, it is essential to consider streamflow at a finer temporal resolution. The commonly used resolution is a calendar day. This finer resolution is important for assessing the frequency of failure of a scheme (for example a direct abstraction), the ecological impacts of flow derogation and, in the case of discharges, the impact of flow on water quality and hence the flora and fauna of the stream. In estimating the time series of natural streamflows at a site for these purposes it is not necessary to exactly replicate all aspects of the true natural time series. Rather, the requirement is to simulate important facets of the regime including:

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- an acceptable simulation of mean flow – conservation of mass;
- the recession behaviour of streamflow;
- the correct representation of seasonal patterns within the flow regime;
- the correct streamflow response to precipitation and the dependencies of that response on antecedent catchment conditions.

With regard to the last point, it is not important to simulate accurately individual high flow events. The only restrictions on the modelling of high flows are that mass should be conserved over a longer time period and the observed sequencing of high flow events should be replicated. In the context of run-of-river schemes, the high flows are not a resource that can be readily utilised due to problems with debris, high concentrations of suspended solids and other quality problems. The uncertainty in our knowledge of the cause-effect links between flow and habitat for aquatic species means that current predictive methods for assessing the ecological impacts of high flows are not sensitive to the absolute magnitude of the flows.

The science of relating hydrological phenomena to physical and climatic characteristics of a catchment, or region, is commonly called regionalisation. There are many examples within the international literature of studies directed at the regionalisation of rainfall-runoff models (e.g. Abdulla, 1997a,b; Agung *et al*., 1995; Sefton and Boorman, 1997). The majority of these studies (but not all) have concentrated on flood estimation or the prediction of monthly and/or annual-resolution streamflow data. Experience has shown that, if a study into the regionalisation of a daily conceptual rainfall-runoff model is to be successful and substantive, the following aspects must be addressed.

- The model must be applied to a representative sample of catchment types across the country.
- In catchment specific applications, the model must be able to simulate accurately those aspects of the flow regime that are of importance for water resource assessment and management decisions – and this must hold over the full range of catchment types.
- The calibrated parameter vectors must be identifiable for specific catchment types, (within the uncertainty of the input data), and the model-fit must be stable when applied to an independent evaluation period.
- The model structure, and hence the parameters, must have physical meaning to enable the parameters to be estimated from the physical characteristics that can be used to differentiate between catchment types.

METHODOLOGY

The fundamental approach to the research was to calibrate parameter-sparse models against streamflow within a large, representative sample of catchments and to

to relate the model parameters to the climatic and physiographic characteristics of the catchments. The extensive literature on this subject has shown that the parameters of a calibrated model are a function of the:

- input data (including observed stream flow) and errors within the input data;
- model structure;
- structure of the catchment being modelled;
- calibration scheme (including the selection of objective functions).

The study was structured as a logical progression from the evaluation and development of techniques for estimating requisite climatic input data, through model calibration and regionalisation of model parameters, to the final evaluation of the success in meeting the overall research objective.

Within the study two model structures were considered; termed MODA and MODB. Both were constructed using the Probability Distributed Model (PDM) toolkit of Moore (1985 and 1999). Both structures utilised the concept of a probability-distributed representation of soil storage capacities (MODA a Pareto distribution and MODB a uniform distribution) and both models used a second-order linear routing reservoir scheme for simulating quick and slow flow routing of effective rainfall. MODA, the more complex model, included an interception storage term and a soil moisture related drainage term, and had five free parameters for calibration. These were three parameters related to the loss module and the two time constants for the routing reservoirs. MODB had one parameter for the loss modules, employed a fixed partitioning of effective rainfall between quick and slow flow routing paths (based on the soil classes within each catchment) and thus had three free parameters for calibration. MODB was developed to minimise the issue of parameter covariance between the parameters of the loss module. The conceptual structure for MODB is presented in Figure 1 and the parameters for the model are described in Table 1. The discussions within this paper are restricted to the regionalisation of MODB.

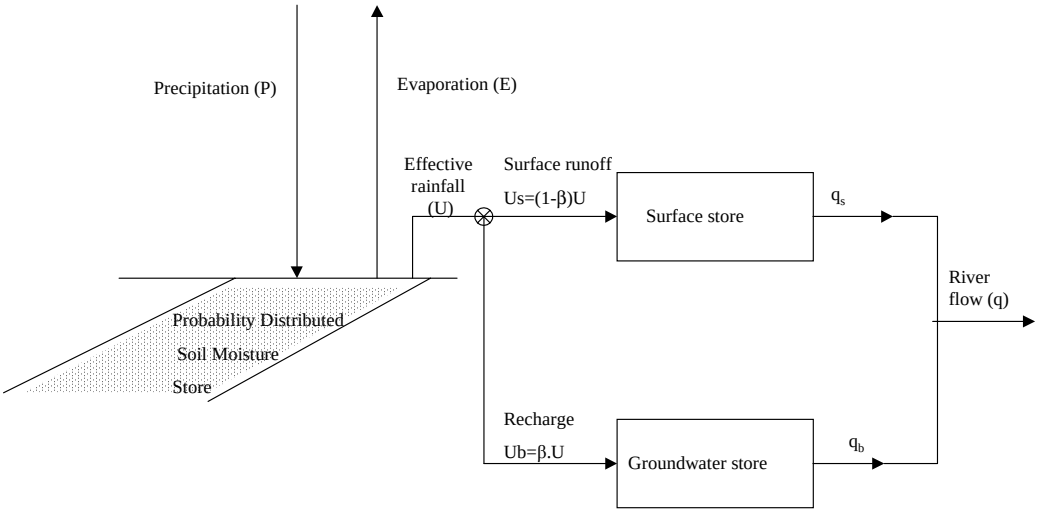


Figure 1. Diagrammatic representation of conceptual structure for MODB

Table 1. Model parameters

Parameter	Unit	Description
Cmax	mm	The maximum store capacity within the catchment
β	none	The runoff factor, which controls the split of direct runoff between surface and groundwater storage routing for MODB
K1	hour	The time constant for the quick flow linear reservoir
Kb	hour	The time constant for the slow flow, or base flow linear reservoir

The catchment data set for the study was constructed by selecting good hydrometric quality, natural, gauged catchments from the National River Flow Archive held at CEH-Wallingford using the classification system of Gustard *et al*. (1992). This classification was informally updated though consultation with the Environment Agency and the Scottish Environment Protection Agency, particularly with respect to the degree of artificial influence. On this basis 318 candidate catchments were selected of which approximately 200 were used within the study. A comprehensive set of catchment characteristics and descriptors was derived for these catchments including FEH topographic descriptors of the catchment and stream network within the catchment (Bayliss, 2000), a range of climatic regime characteristics and other characteristics describing the catchment soils (Boorman *et al.*, 1995) and vegetation classes (Fuller *et al.*, 1994).

Daily time series of catchment average precipitation were derived for each catchment using the Meteorological Office daily rainfall library and a modified version of the Triangular Planes interpolation methodology of Jones (1983). Time series of Potential Evaporation (PE) were estimated for each catchment using a scheme for dis-aggregating Meteorological Office of Rainfall and Evaporation Calculation System (MORECS) II Penman-Monteith PE estimates from a 40 km to a 1 km resolution grid, taking into account the impact of increased spatial heterogeneity of topographic controls on PE.

The classical approach to calibrating a rainfall-runoff model commonly employs the use of an automatic algorithm to search a feasible parameter space coupled with manual intervention. Manual intervention enables the hydrologist to make decisions about the trade-off between different aspects of model-fit, based on experience. The calibration scheme developed for this study was a multi-criteria, constrained, random walk scheme that utilised a number of objective functions, allowed for trade-offs between different aspects of model-fit and recognised that the input streamflow and climatic data have an associated uncertainty.

Within a catchment, the scheme is used to identify a large number of parameter sets, or vectors, that can be considered equally likely given these constraints. One model parameter vector was selected from the many candidate vectors for the regionalisation analysis. The selection was based on both the performance of the model when used with each vector and the stability of that performance when the model was applied to an

independent evaluation period. A combined Euclidean objective function was developed for this selection. The components of the function assessed the model-fit from the perspective of closing a water balance, the general time series structure of the data and the fit over recession periods, with a particular emphasis on low flows. The selection of a best compromise parameter vector reflects that a good, stable model-fit is more likely to be a function of the catchment type than one that just gives a good fit over a calibration period, and hence may be incorrectly specified. An evaluation of the calibrated model performance identified that the model simulated streamflow extremely well in the majority of catchments and that these model-fits were generally stable when applied to an evaluation period.

A number of approaches, based on multi-variate regression analysis and nearest neighbour methods, were adopted for the regionalisation of the calibrated model parameter sets, or vectors, for the catchments used in the study. The most successful of these was the development of multi-variate regression-based relationships between model parameters and catchment characteristics/descriptors – and the results presented within this paper are for this approach. The objective was to identify robust, conceptually supportable, relationships. Extensive split-sample tests were used to demonstrate that the regression models were not over specified, and hence were independent of the catchment sample used to develop the models.

EXAMPLE RESULTS

The performance of the models, when used with the regionalised parameter estimates was extensively evaluated. Two examples of these results are presented here. The first of these is a classification of the ability of the regionalised model to (i) close an adequate water balance (represented by the modulus of the percentage error in the estimation of mean flow (BIAS)) and (ii) maintain a reasonable simulation of the hydrograph fit (using the Nash-Sutcliffe efficiency criterion (EFF)). The range of EFF is between 0 and 1, a value of 1 represents a perfect fit and a value of zero indicates that the model fit is no better than that which would be obtained if the simulated flows were replaced by the mean of the observed flows. The class intervals were based on four main BIAS classes and two EFF classes. The main BIAS classes were:

- Class A: model BIAS of less than 5%, equivalent to a good quality gauged flow record;
- Class B: model BIAS between 5 and 10%, the 10% level is equivalent to the acceptable artificial influence allowed during catchment selection;
- Class C: model BIAS between 10-20%, approximately equivalent to the accuracy of a Micro LOW FLOWS estimate of mean flow;
- Class D: model BIAS greater than 20%, classed as an unacceptably large BIAS error.

Micro LOW FLOWS is a software package used by regulators within the UK for estimating flow statistics within ungauged catchments (Young *et al.*, 2000). The first three BIAS classes were subdivided into two EFF classes: a good fit class (Class 1) in

which EFF was greater than 0.6 and an acceptable fit class (Class 2) in which EFF was between 0.5 and 0.6. One additional class, Class E, was used for catchments in which the model failed to explain more than 50% of the variance (EFF<0.5), irrespective of the

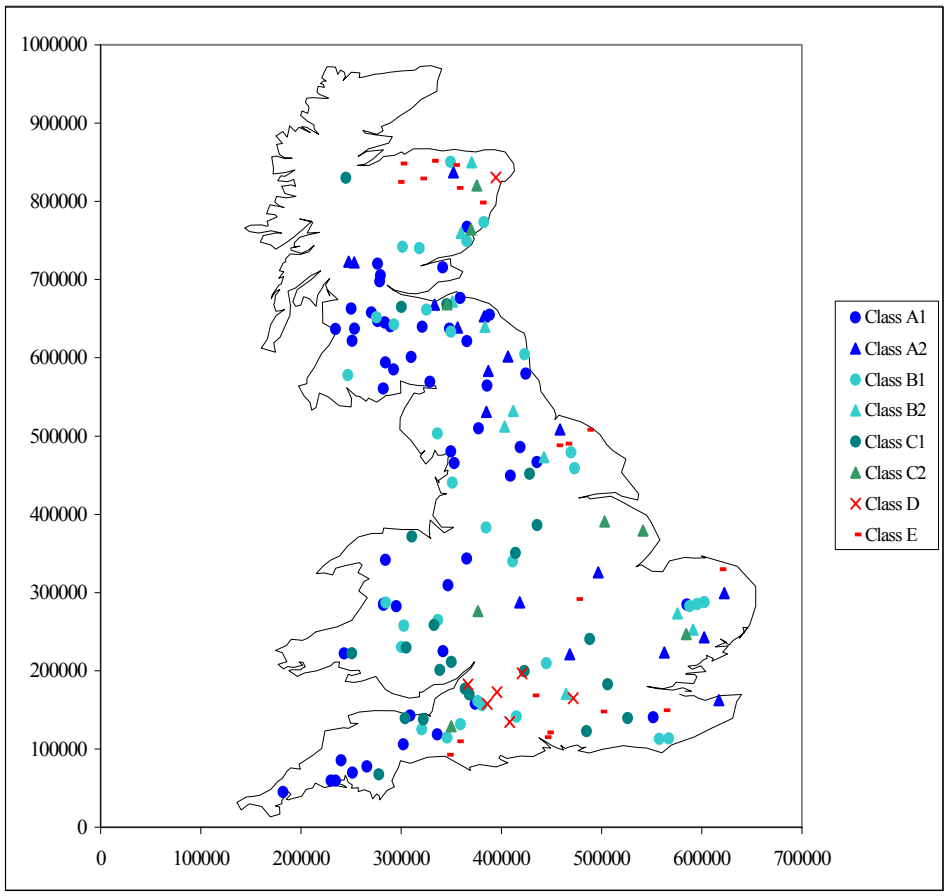


Figure 2. The spatial distribution of model fit classes across the UK

value of BIAS. The results of this classification for the evaluation period for catchments used in the study are presented spatially in Figure 2.

Simulations that were good, or very good, were obtained within 67% of catchments (Bias Classes A and B) and simulations that were acceptable, or better (Bias Classes A, B and C) were obtained within 85% of catchments. Catchments in which an acceptable water balance could not be closed tended to be located in the south of the country, and were dominated by groundwater catchments. Catchments in which the time series model-fit was judged to be unacceptable tended to be dry catchments on the east of the country. Inspection identified that the raingauge network tended to be sparse over these catchments, potentially resulting in poor estimates of precipitation inputs to the models. The time series criterion for judging the acceptability of the simulation was harsh. The simulations for many catchments that failed on this criterion were able to close a water balance and simulated periods of recession well. The number of catchments that had completely unacceptable time series fits, from a water resources perspective, was considerably lower that the classification suggests.

The second set of results presented for illustrative purposes are the 1988 observed and simulated hydrographs for a small, low rainfall surface water catchment, the Pinn at Uxbridge (Figure 3). The simulated hydrographs are those obtained with the calibrated model parameters and the regionalised model parameters. In the catchment classification scheme above, this catchment would lie in Class A1 for the calibrated parameters and C1 for the regionalised parameters.

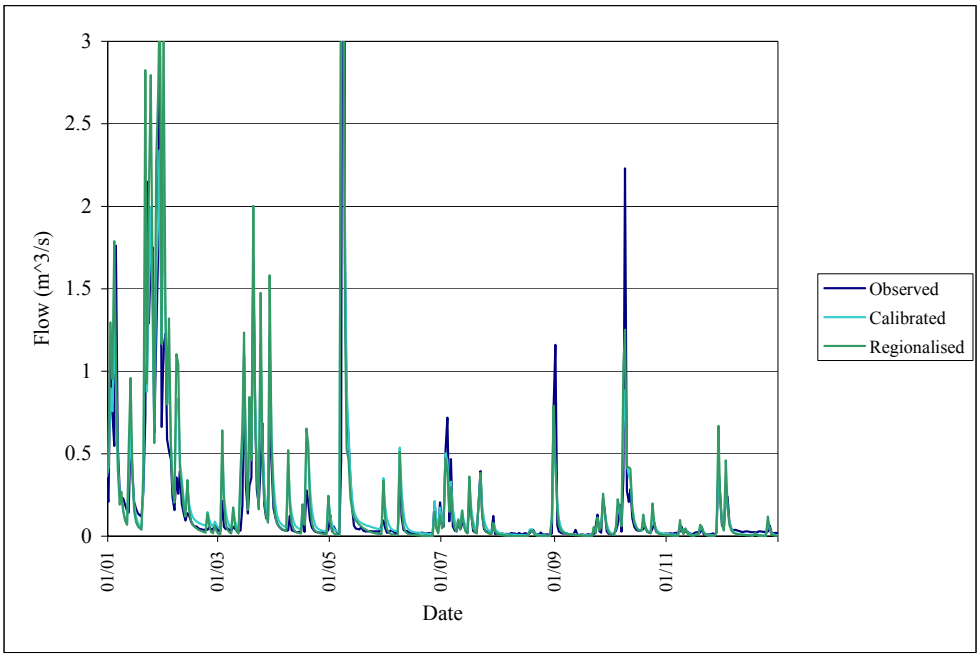


Figure 3. Example observed and simulated hydrographs

CONCLUSIONS AND ACKNOWLEDGEMENTS

The paper has provided a brief overview of what was a large study. From the potential for use as a low-cost alternative to the current practice of re-scaling gauged flow records from analogue catchments, the generally good performance of the regionalised models is very encouraging. Research is ongoing at CEH and it is hoped that it will lead to a suite of "desktop" tools in the future. The research presented within this paper was funded by the Environment Agency and the Centre for Ecology and Hydrology on a shared cost basis.

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Classifying components of river basins for hydrological model parameter estimation

Anne F.J. Reungoat and William T. Sloan

INTRODUCTION

Hydrological models invariably require some element of calibration using hydrological data. This causes problems when applying models to river basins where no hydrological data exist (ungauged basins), a task that hydrologists are increasingly being called upon to undertake. Most attempts at resolving these are based on "regionalisation" techniques traditionally used in regional flood frequency analysis to infer river flow statistics for ungauged river basins from those at gauged rivers basins. When applied to estimate model parameters, these techniques have unfortunately met with limited success. In this case, they involve three stages. Firstly, a classification stage which consists of grouping together the river basins whose hydrological regimes are likely to be similar. Secondly, flow data from gauged river basins of each group (region) are used to parameterise the hydrological model, and regression equations relating the physical and climatic river basin characteristics and model parameters (instead of the river flow statistics) are determined. The final stage consists in applying the established regression equations to infer the model parameters in the ungauged river basins of each region. In this process, the classification stage is critical and has also proved problematic. The main problem arises from the fact that seeking groups of river basins that are homogeneous across all components of their hydrological regimes leads to a large number of groups, each of which contains only a few basins. It is therefore difficult to have confidence in the regression equations derived for inferring model parameters, as they rely on very little data. An alternative approach, which has been suggested by some researchers, is to relax the degree of homogeneity sought in each group, thus creating larger but less homogeneous groups. Again, it is difficult to have confidence in the regression equations derived from such groups since they have to account for more of the variability in model parameter values using basin characteristics. Studies conducted by Abdulla and Lettenmaier (1997), and Sloan (1999), independently report significant errors in regional modelling studies of the same area in central USA. They both attribute this to the fact that regionalisation of model parameters had been conducted within heterogeneous groups of sub-basins.

Therefore, if we are to have confidence in hydrological model results for ungauged river basins, there is an urgent need for a pragmatic method for classifying river basins into hydrologically homogeneous groups.

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This paper outlines a new method for identifying regions, which differs from other currently available methods in two significant ways. The first is to discard the constraint that each basin only belongs to one group, and to propose that each basin undergoes three independent classifications, one relative to each of the major components of the land phase of the hydrological cycle: the interaction soil, vegetation and atmosphere; surface flow; groundwater flow. The second difference is in the methodology for objectively selecting which of the basin characteristics are the most appropriate classifiers. This paper focuses on the development of the "surface-flow related" classification. A new approach to classifying river networks according to their flow regime is proposed. It uses variables that can be derived easily from data stored in digital databases. For example, variables describing the topology of the network that have been derived automatically from digital Ordinance Survey maps. The objective method for selecting the best classification variables uses a comprehensive 1-D river network model within a Monte-Carlo framework.

CLASSIFYING RIVER BASINS

The ultimate aim of the research presented in this paper is to devise a river basin classification method specifically designed for estimating hydrological model parameters in ungauged river basins. The proposed river basin classification scheme differs from previously used schemes (e.g. Bates *et al*. (1998), Post and Jakeman (1996), Hall and Minns (1999)) in two significant ways. The first, is to discard the constraint that each basin only belongs to one group, and propose that each basin undergoes three independent classifications, one for each of the major components of the land phase of the hydrological cycle: interaction of soil, vegetation and atmosphere; surface flow; and groundwater flow. The advantage of this approach is that group sizes can remain fairly large without compromising homogeneity. Consequently, there will be increased confidence in the regression equations derived for each group. The second difference is in the methodology used for objectively testing which of the basin characteristics, from a set of potential classifiers, are the most appropriate.

The set of classifiers is constrained by excluding climatic characteristics, which are used as input data to hydrological models. It is further constrained by the fact that "component related" classifications are conducted so that fewer characteristics are of importance in describing the hydrology of one river basin component (e.g. surface water flow) than the hydrology of the basin as a whole. However, there still remains a large number of potential classifiers and it is necessary to objectively test the relative importance of each of them. Such a test would normally require measured hydrological data. Herein lies the major difficulty and, consequently, the major research issue in partitioning river basins into component parts for classification.

It is almost impossible to measure hydrological variables that can characterise the components (e.g. surface water) in isolation. However, sufficiently comprehensive hydraulic and hydrological models describing individually the river basin components do exist to allow synthetic data to be generated. Therefore, synthetic data are used to test the relative importance of the various pre-selected classifiers. The proposed objective test consists of Monte Carlo simulations whose results are analysed using standard statistical clustering methods.

CHARACTERISING RIVER NETWORKS

Although the research outlined here includes classification schemes for the three major components described by standard hydrological models, the paper focuses on the derivation of a classification scheme for estimation of the surface flow hydrological

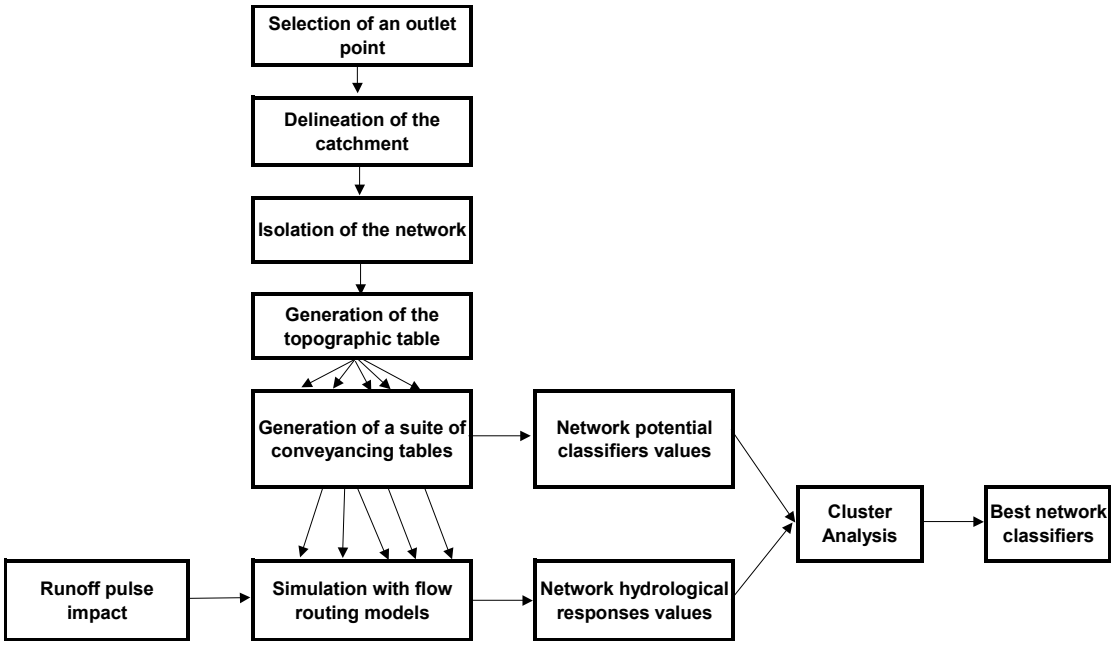


Figure 1. Schematic diagram of the best classifiers objective test

model parameters. A practical method for objectively identifying the variables that characterise river networks is developed. This method involves three stages: first, is the generation of a large set of river network models; second, is the derivation of synthetic data, using a flood routing model to simulate the hydrological response at the outlet of each river network to an instantaneous pulse of runoff distributed uniformly throughout the network; third, cluster analysis is applied to the synthetic data derived by simulation in order to identify variables which best characterise the hydrological response. A schematic diagram of the method for identifying the best classifiers is given in Figure 1.

Generation of river network models

The construction of a model for flow routing through a river network commonly requires topographic and conveyancing data. Topographic data are increasingly becoming available on digital databases. However, determining the conveyance normally requires detailed surveys of river cross-sections. Collecting such data in the field is a difficult and expensive task. The construction, through the use of field data, of a large set of river network models is therefore unfeasible. To overcome this problem, synthetic model parameters are sampled from distributions of physically reasonable values. However, the aim is to keep the synthetic data to a minimum. Therefore, only the river model parameters that determine conveyance are sampled from statistical distributions. These

distributions are constrained by using the rules of hydraulic geometry for stable channels. All the other parameters are derived from real topographic information.

Digital terrain and river networks maps of Scotland were imported into an Arcview GIS database. A suite of programs automatically generates all the topographical information on the channels upstream of a point selected interactively by the user. These programs first delineate the river basin draining through the selected point, then isolate its associated river network, and finally export a table containing a description of the nodes and reaches suitable for the flow routing model. The table of topographic information is augmented by synthetic conveyancing data for each reach of the network. The synthetic data comprises width, depth and Manning's n for a rectangular channel. Approximate values for top width and depth can be calculated using Leopold and Maddock's (1953) power laws

$$W = a\,Q^{f} \qquad (1)$$
$$D = b\,Q^{c}$$

where W is the top width, D is the depth, Q is the mean annual flow and a, b, c and f are constants. These are empirical formulae and, in practice, the values of a, b, c and f vary within bounds for UK rivers (Hey, 1982). Therefore, rather than defining only one possible cross-sectional geometry for each reach, a range of possible cross-sections are assumed to be equally likely. These are derived by randomly selecting values, for the constants in equation (1), from uniform distributions with realistic bounds. Similarly Manning's n values are randomly selected. Therefore, for each river network an ensemble of tables is generated, each with different conveyancing characteristics.

Simulation

The success of the method is partly dependent on the choice of the flow routing model for simulating the response of a network to a pulse of runoff. This model must be capable of dealing realistically with spatially distributed information on physical properties, and have a proven track record of being able to accurately reproduce a network outlet flow when appropriately parameterised. The simulations are conducted using two routing models. In the headwaters of the network, if the channels are steep, flow is routed using a kinematic wave model. Further downstream, where the channels slope more gently, the Saint Venant equations are solved using the NOAH routing model developed at the University of Newcastle, which is computationally efficient and sufficiently robust for use within a Monte Carlo framework. The runoff into each reach of the network is assumed to arrive as a step pulse whose magnitude is proportional to the one year return period rainfall. A range of pulse lengths and consequently storm duration will be applied. The pulse is superimposed on the mean annual flow rate and distributed according to the fraction of the catchment area draining through the reach. In constructing such a runoff hydrograph, the aim is not to reproduce any one particular storm event but rather to have a hydrograph with standardised shape and typical flow rates with which to compare the hydrological responses at the outlet of many river networks.

Cluster analysis

The hydrological response of the river networks to a pulse of runoff is summarised by two simple variables; the percentage of attenuation of the flood peak and the time-to-peak for equal catchment areas. Ultimately, the aim is to identify variables that describe the physical properties of the catchment and best characterise the hydrological response of the network. This is achieved using a standard cluster analysis technique usually referred to as the "Region Of Influence" (ROI) approach (Bates *et al*., 1998)

CONCLUSIONS

The ability to classify river basins into relatively homogeneous groups is central to the successful application of hydrological models in areas where hydrological data are sparse. It allows regression relationships to be derived for inferring model parameters in ungauged river basins where no data exist for calibration. A novel approach to river basin classification is currently being researched at the University of Glasgow. This approach differs from other currently available methods in two significant ways. The first is to discard the constraint that each basin only belongs to one group, and propose that each basin undergo three independent classifications, one relative to each of the major components of the land phase of the hydrological cycle: interaction of soil, vegetation and atmosphere; surface flow; groundwater flow. The second difference is in the methodology for objectively selecting which of the basin characteristics are the most appropriate classifiers. It is anticipated that this new approach will significantly improve the reliability of regression equations used to infer model parameters in ungauged catchments.

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On the evaluation of conceptual rainfall-runoff models using multiple objectives and dynamic identifiability analysis

Thorsten Wagener and Howard S. Wheater

INTRODUCTION

Although continuous simulation models of river flows have been available for several decades, a major limitation has been the inability to model flows on ungauged catchments. This is partially a reflection of ambiguity in the model structure and parameter values associated with conceptual hydrological models. Various parameter sets, often widely distributed within the feasible parameter space (Wagener *et al*., 2001a) and sometimes even different conceptualisations of the hydrological system (e.g. Uhlenbrock *et al*., 1999) can yield equally good results in terms of a predefined objective function. In addition, due to structural inadequacies, a single parameter set might not be adequate to describe all response modes of a hydrological system within a specific model structure (Boyle *et al*., 2000; Wagener *et al*., 2001a). This ambiguity leads to parameter and consequently predictive uncertainty (Beven and Binley, 1992), and therefore limits the applicability of conceptual models, *e.g.* for the simulation of land-use or climate-change scenarios, or for regionalisation studies.

One reaction to this problem of ambiguity of system description is the search for calibration methods with higher discriminative power (Gupta, 2001). Such methods aim to make better use of the information contained in the available time-series and also to indicate structural inadequacies. They therefore provide an objective analysis of the suitability of a model (*i.e.* a specific parameter set within a specific model structure) and a model structure, respectively. Calibration is seen here as the process of evaluating models and model structures (including all possible models within this structure) with respect to their suitability to represent the hydrological system under investigation. Competing model structures can be analysed in order to establish preferences between models, or, in case of severe underperformance, to reject certain models or model structures completely. The analysis of individual structures can be used to evaluate whether underlying assumptions are violated, with the aim to either corroborate or reject a specific structure. This approach is in line with a scientific method of hypothesis testing as proposed by Popper (1999), *i.e.* the testing of model structures, and the rejection of those that fail to perform.

An example application is shown based on rainfall-runoff modelling of UK catchments where the approaches described above are used in a framework to evaluate different, relatively simple model structures containing common structural elements.

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MODEL IDENTIFICATION AS AN EVALUATION PROCESS

Evaluation of competing model structures – multiple objectives

The comparison of competing model structures is traditionally done using measures of performance that aggregate model residuals over the whole period of available data. While this is an important aspect of performance testing, it also has some serious drawbacks with respect to the loss of information (e.g. Boyle *et al*., 2000; Wagener *et al*., 2001a). Multi-objective evaluation frameworks have therefore been proposed (Gupta *et al*., 1998). One approach to improve the use of information by defining different objective functions is the segmentation of the hydrograph into different response modes, therefore sometimes even allowing the allocation of performance to a particular model component (e.g. Wagener *et al*., 2001a). One model structure might for example contain the better soil moisture accounting component, while another might have a superior slow response routing component. Finding these superior components would allow the modeller to combine them to a new and better-suited model structure.

Additionally, it can be shown that the number of parameters that can be confidently identified using a single objective function is rather small, around 3-5 (Gupta, 2001). Wagener *et al*. (2001a) show that a multi-objective approach can increase the identifiability of individual parameters considerably.

A multi-objective approach is therefore required to allow a detailed evaluation of competing model structures.

Evaluation of individual model structures – dynamic identifiability

The analysis of the (in)adequacy of model structures has received increasing interest in the research literature (e.g. Gupta *et al*., 1998). This is caused by the awareness that model structural error constitutes a large part of the overall uncertainty in the modelling process.

Model structures are usually selected based on the hydrologist's understanding of the hydrological system under investigation; it is therefore a subjective decision. The positive confirmation of this selection, *i.e.* the conclusion that the 'true' model structure has been found through e.g. a verification step, is logically difficult to justify (Popper, 1999). It can only be said that a selected model structure is capable of reproducing the available time-series in an acceptable manner. New measurements, possibly including more extreme conditions, might still lead to the failure of the model. However, the contrary, the rejection of unsuitable model structures, is possible (e.g. Wheater *et al*., 1993; Beven, 2000).

One approach is to test the assumptions underlying the selected model structure. A common assumption is that the model parameters are, although not measurable in the field, constants in time and representative for some inherent property of the hydrological system (Bard, 1974). Beck (1985) showed how this assumption can be analysed using the Extended Kalman Filter (EKF). However, the method includes a range of limiting assumptions and the interpretation of the results is often difficult (Beck, 1987).

A new approach, based on the well-known Regional Sensitivity Analysis (RSA, Spear and Hornberger, 1980) can be applied to, amongst other things, evaluating the suitability of model structures in a more robust manner (Wagener *et al*., 2001d; Figure 1). The procedure uses a uniform random sampling method and the resulting parameter population is conditioned on a measure of performance, similar to the RSA extension introduced by Beven and Binley (1992) in their Generalized Likelihood Uncertainty Estimation (GLUE) approach. However, the performance is not calculated utilising the residuals over the complete calibration period, but it is rather evaluated as a running mean, using a range of window sizes. A new measure of identifiability, introduced by Wagener *et al.* (2001a), is used to derive the degree of identifiability of individual parameters from the conditioned populations. The measure uses the gradient of the (conditioned) cumulative parameter distribution, with a higher gradient for more peaky distributions (Figure 1(b) and (c)). Applying the measure in a dynamic form, called Dynamic Identifiability Analysis (DYNIA, Wagener *et al*., 2001d), one can identify periods where specific parameters are identifiable or not, and the deflection of 'good' parameter locations in time can be utilized as an indicator of structural problems.

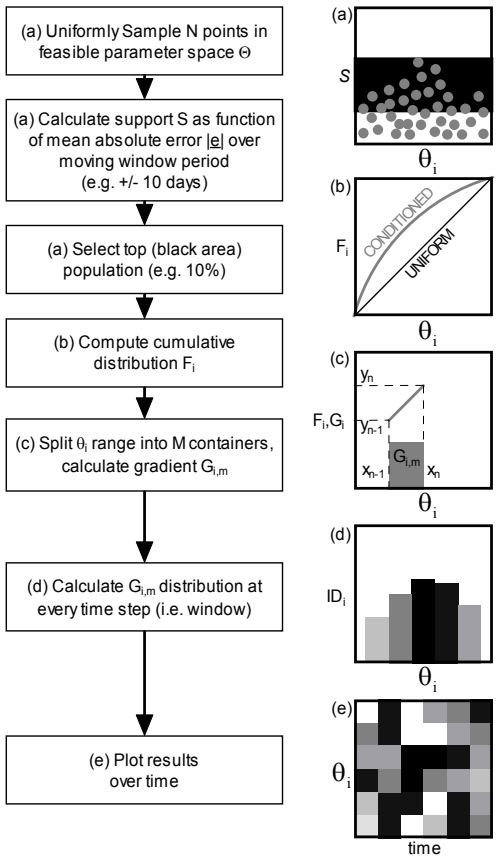


Figure 1. Dynamic Identifiability Analysis procedure.

Aggregating the residuals over a period of time (e.g. the calibration period) into a measure of performance such as the root mean squared error leads to a loss of information. DYNIA is an attempt to avoid this loss and give additional insight by aggregating the residuals in form of a running mean, over different window sizes. This can for example highlight structural inconsistencies. Of course, this is only the first step to derive an improved model structure; the steps of finding the cause of the failure and then developing an improved model structure are again dependent on the experience and creativity of the hydrologist.

CASE STUDY

Data and modelling tools

The results shown are derived using data from the Lower Medway at Teston (1256.1 km^2) located in south-east England. Six years (10/04/90 – 14/07/96) of data (daily naturalised flows, precipitation, potential evapotranspiration and temperature) are available. The geology of the Medway catchment is characterised by a mixture of permeable (chalk) and impermeable (clay).

The Rainfall-Runoff Modelling Toolbox (RRMT) and the Monte-Carlo Analysis Toolbox (MCAT), developed at Imperial College, are used for calculation

and visualisation of results (Wagener *et al.* 2001b). The RRMT is a generic modelling shell allowing its user to implement different lumped model structures of conceptual or hybrid metric-conceptual type. The MCAT combines a number of visualisation and analysis tools to investigate parameter behaviour and modelling uncertainty.

All model structures implemented in the RRMT consist of a soil moisture accounting (SMA) and a routing component. A variety of popular SMA modules are compared here. The SMA modules used to produce effective rainfall are Catchment Moisture Deficit (CMD, Evans and Jakeman, 1998), Probability Distributed Model (PDM, Moore, 1999), and Penman (PEN, Penman, 1949) type structures (Figure 2).

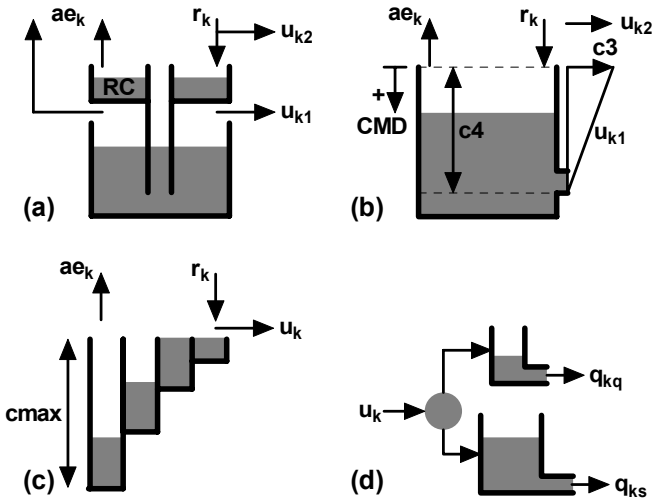


Figure 2. SMA model structures and routing module: (a) Penman two-store (PEN), (b) Catchment Moisture Deficit (CMD), (c) PDM, (d) routing module for effective rainfall (u), from each SMA.

The PDM structure utilized here produces effective rainfall by an overflow mechanism only and the evapotranspiration is at the potential rate while water remains available. In all cases investigated here the routing component used to translate the effective rainfall into streamflow is a parallel structure of linear conceptual reservoirs.

Methodology

The multi-objective (MO) analysis is based on a random sampling procedure selecting 20000 parameter sets from uniform distributions for each parameter. The dynamic analysis is based on a smaller sample size of 2500 parameter sets, derived in the same manner.

Five different objective functions (OF), all based on the root mean squared error, are defined for the MO analysis. The first is an overall measure (RMSE), using the whole time-series available. The remaining OFs use periods selected based on the hydrograph slope and a simple threshold value (Figure 3). Two OFs use only the residuals during recessions, *i.e.* periods of a falling hydrograph. A simple threshold value (the mean of the flow) is used to separate quick and slow recession periods, FQ and FS. Two further OFs describe the rising periods during high and during low flows, FDH and FDL. These OFs are based on the assumption that different processes are dominant during periods of high and periods of low flow, and during periods of

catchment wetting up and drainage. The root mean squared error for the different models and the identifiability measure for all parameters are calculated for all the periods.

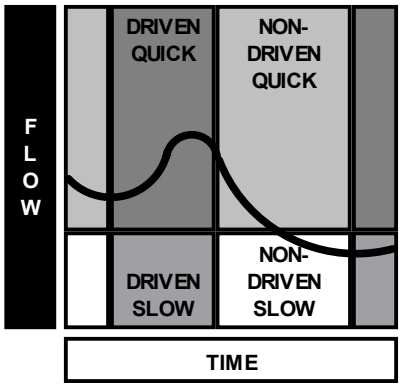
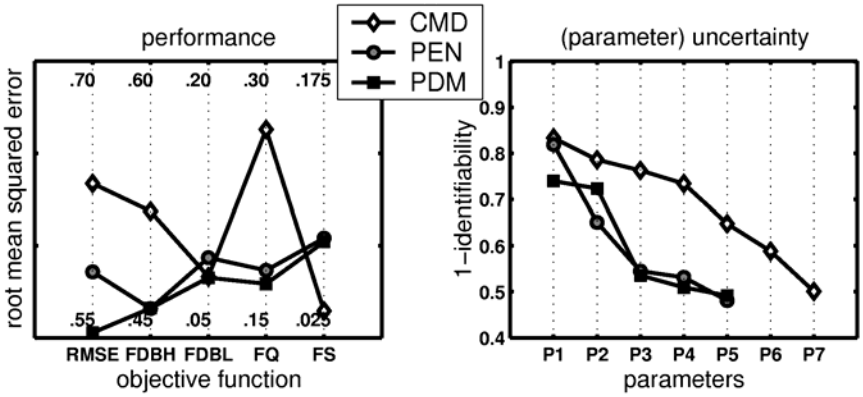


Figure 3. Segmentation based on hydrograph slope and a threshold flow



Results and discussion

The results of the MO analysis are shown in Figure 4. It shows the performance of the best parameter sets for each model on the left and the identifiability of the model structures parameters on the right. Better values are located towards the bottom of both plots. One can see that the Penman and PDM type model structures are superior to the CMD one. However, the drainage (bottom) outlet of the CMD structure allows it to simulate the system behaviour during low flow periods (FDBL and especially FSB) more accurately. This information is hidden by the aggregation of the residuals over the total calibration period. The CMD structure also shows a lower identifiability of its parameters (Figure 4, right).

The results of the DYNIA procedure for one example parameter are shown in Figure 5. The b parameter, which describes the shape of the Pareto distribution of storage capacities in the PDM model structure is analysed. One can see that the confidence limits (grey lines) narrow during periods of low flow (black line). The black shadings indicate locations in the parameter space of better performing parameters. This parameter is unidentifiable during periods of high flow.

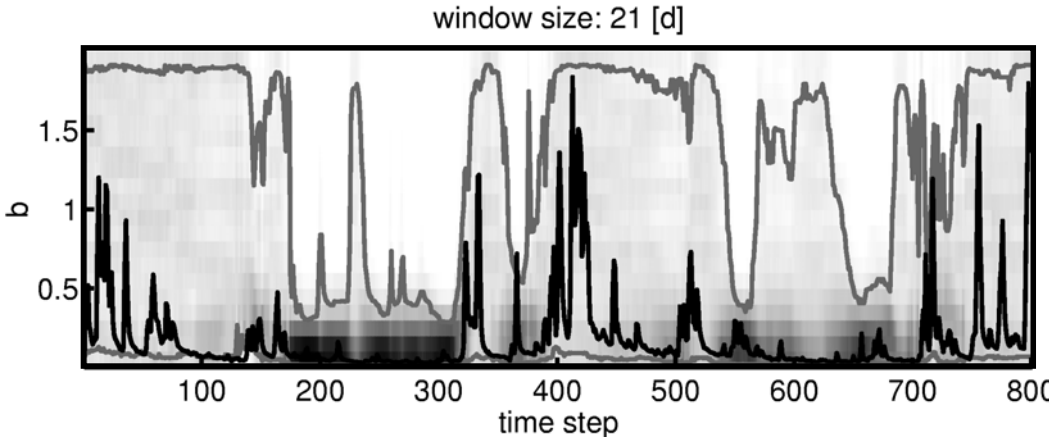


Figure 5. DYNIA result for parameter b of the PDM model structure

CONCLUSIONS

This paper introduces a new framework for the evaluation of hydrological models and model structures. The analysis is based on two approaches, a multi-objective analysis of competing models and model structures, and a new dynamic identifiability approach to evaluate individual structures.

Conclusions so far can be summarized as follows. The multi-objective approach has been shown to be advantageous in terms of providing additional information to, amongst other things (Boyle *et al*., 2000; Wagener *et al*., 2001a): (1) allow comparison of structural components, not just complete structures. (2) give a more objective picture of the identifiability of individual parameters and the model's performance.

The dynamic identifiability approach has already been successfully used to (Wagener *et al*., 2001c;d): (1) analyse individual model structures, (2) identify suitable parameters in an approach more in line with sophisticated manual approaches. (3) Identify periods of high information content for individual parameters, for example as a first step in a multi-objective analysis, and (4) analyse the suitability of experimental design, e.g. tracer experiments to identify model parameters.

These approaches can be combined to form a scientific framework of model evaluation (Wagener *et al.*, 2002) in line with a scientific method as advocated, for example, by Popper (1999).

ACKNOWLEDGEMENT

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Flood frequency estimation by continuous simulation (with uncertainty)

David Cameron, Keith Beven, Jonathan Tawn and Pamela Naden

ABSTRACT

This paper explores several important hydrological modelling topics surrounding the use of continuous rainfall-runoff simulation for flood frequency estimation. A continuous simulation methodology suitable for flood frequency estimation is described. The methodology features a rainfall-runoff model (TOPMODEL), a new profile-based stochastic rainfall model, and an uncertainty estimation procedure (Generalised Likelihood Uncertainty Estimation, or GLUE). By explicitly accounting for a catchment's soil moisture conditions, allowing the direct simulation of long return period flood events (via the coupling of TOPMODEL with the stochastic rainfall model), and quantifying the uncertainty associated with the simulated flood estimates, this methodology is an attractive alternative to the more traditional statistical and event-based techniques available for flood frequency estimation. Example applications to gauged, UK catchments are provided. These include a consideration of the impacts of climate change upon flood frequency, as assessed under the methodology.

INTRODUCTION

With more available computing power and sub-daily rainfall and flow data in digital form, it is becoming possible to move towards standard techniques for flood hydrograph estimation based upon the continuous simulation of flow. By using a continuous rainfall series (either observed or generated via a stochastic rainfall model), running a hydrological model continuously and going on to analyse the flood peaks of the simulated flow series, it is no longer necessary to make the assumptions behind the design event approach or the simplifying assumptions of the event-based derived-distribution approach pioneered by Eagleson (1972). Furthermore, as soil moisture accounting is dealt with automatically, assigning an antecedent wetness condition is no longer a problem. While this method has yet to be fully proven, initial results are promising (Beven, 1987; Calver and Lamb, 1996; Naden *et al*., 1996; Blazkova and Beven, 1997; Lamb, 1999). The use of continuous simulation also opens up several areas of research interest. These include: the potential impacts of climate change upon flood frequency; the choice of stochastic rainfall model; and the possible consistency between continuous flow simulation and flood frequency estimation rainfall-runoff model parameterisations.

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In what follows, a continuous simulation methodology suitable for flood frequency estimation is described. The methodology features a rainfall-runoff model (TOPMODEL), a new profile-based stochastic rainfall model, and an uncertainty estimation procedure (Generalised Likelihood Uncertainty Estimation, or GLUE). Example applications to gauged UK catchments are provided. These include a consideration of the impacts of climate change upon flood frequency, as assessed under the methodology.

METHODOLOGY

The hydrological model

TOPMODEL is a simple semi-distributed model of catchment hydrology that predicts storm runoff from a combination of variable saturated surface contributing area and subsurface runoff (e.g. Beven 1986, 1987; Quinn and Beven, 1993). The dynamics of the contributing area for rapid runoff as the catchment wets and dries are based on a quasi-steady state analysis. The topographic index ln (a/tanβ) is used as an index of hydrological similarity, where a is the area draining through a point, and tanβ is the local surface slope. The use of this form of topographic index implies an effective transmissivity profile that declines exponentially with increasing storage deficits. Evapotranspiration losses are controlled by potential evapotranspiration and storage in the root zone with the parameter *SRMAX* (effective available water capacity at the root zone).

For the purposes of initial flood frequency estimation, TOPMODEL is driven by observed hourly catchment rainfall inputs. Longer return period estimates are then obtained through the coupling of TOPMODEL with a stochastic rainfall model.

The stochastic rainfall model

Full details of the stochastic rainfall model are provided in Cameron *et al*. (1999, 2000a, 2000c). The model's performance compares favourably with other profile-based and pulse-based stochastic rainfall models (Cameron *et al*., 2000c, 2001).

The stochastic rainfall model is based directly on the observed hourly rainfall record available at a site and generates random rainstorms via a Monte Carlo sampling procedure. The model characterises a storm in terms of a mean storm intensity, duration and inter-arrival time, in addition to a storm profile component. The dependence of mean storm intensity upon storm duration is incorporated into the model through the subdivision of the observed storm event series into duration classes.

For each duration class, an empirical cumulative distribution function (cdf) of mean storm intensity is constructed directly from the events located within that class. Where appropriate, the upper tail of a given cdf is extrapolated via the fitting of a Generalised Pareto distribution (GPD). This is done in order to permit the generation of extreme storm events unrecorded within the available catchment storm series. The storm duration and inter-arrival time characteristics derived from the observed event series of each catchment are also modelled using their empirical cdfs.

The final component of the model is a storm profile. For each duration class, a database of storm profiles is compiled from the observed rainstorm event series (or by data transfer from a similar catchment, e.g. Cameron *et al*., 2000a). These are normalised by cumulative volume and total duration. During a model run, the profiles are randomly selected in order to provide storm profiles for the simulated rainfall events.

The Generalised Likelihood Uncertainty Estimation (GLUE) framework

Every flood frequency estimate is subject to some degree of uncertainty. The major sources of this uncertainty in the continuous simulation approach include the limitations of the observed data series and the choice of rainfall and hydrological models. In the methodology described in this paper, the Generalised Likelihood Uncertainty Estimation (GLUE) framework of Beven and Binley (1992) is used to assess this uncertainty (see also Beven, 1993; Freer *et al*., 1996; Franks *et al*., 1998; Cameron *et al*., 1999).

The GLUE methodology rejects the concept of a single, global optimum parameter set and instead accepts the existence of multiple acceptable (or behavioural) parameter sets (the concept of model equifinality: Beven, 1993). Cameron *et al*. (1999, 2000a) describe the application of GLUE to flood frequency estimation in detail. This procedure can be summarised as follows.

Five thousand TOPMODEL parameter sets, containing a fairly broad range of parameter values, are initially generated from independent uniform distributions. A single continuous simulation of the hourly observed series, utilising observed hourly rainfall inputs, is derived from each parameter set. Maximum likelihood is used to fit separate Generalised Extreme Value (GEV) distributions to the observed and individual simulated series of hourly annual maximum flood peaks. The performance of each parameter set is evaluated using a log likelihood function. Parameter sets are rejected as non-behavioural if they fail to meet a pre-defined threshold criterion (this threshold is set following a period of testing of different possible thresholds and a consideration of the resulting number of parameter sets retained). Behavioural parameter sets are then evaluated in terms of their ability to reproduce further observed flow statistics (e.g. the continuous hourly hydrograph and the flow duration curve).

In order to provide flood frequency estimates beyond the upper return period limit of the observed series, the stochastic rainfall generator is coupled with TOPMODEL. This requires the estimation of the rainfall model's GPD parameters prior to the runs with TOPMODEL and this is also achieved within the GLUE framework. The (combined) likelihood weights of one thousand behavioural (TOPMODEL and GPD) parameter sets are used to produce likelihood weighted uncertainty bounds for the flood frequency curve from simulated hourly time-series of 1000 year lengths. In this evaluation procedure (and other continuous simulation approaches, e.g. Lamb, 1999), it is worth noting that the available observed data series is only one realisation of numerous possible observed series of equal record length (Beven, 2001). Consequently, the resulting flood frequency simulations (which are based upon this available sample of observed data) may not fully represent the underlying population of flood events.

APPLICATIONS

To date, the methodology has been applied (under current climatic conditions) to five gauged catchments in the UK: the Wye, Tanllwyth, Browney, Dowles Brook and Findhorn (Cameron *et al*., 1999, 2000a). Table 1 details the results of the initial constraining runs for the latter four catchments. It can be seen that the strategy is effective in rejecting non-behavioural parameter sets, particularly for the catchments with reasonably long continuous hourly rainfall and flow data. The results also

suggest that there is a reasonable consistency between rainfall-runoff model parameterisations for both flood frequency estimation and continuous flow simulation. Cameron *et al*. (1999) and Lamb (1999) indicate that, currently, it may not be possible to reproduce the exact rankings and timings of an observed flood peak series.

Table 1. Number of TOPMODEL parameter sets (of 5000) retained following rejection of non-behavioural parameter sets

	Tanllwyth	Browney	Findhorn	Dowles Brook
No. years data	16	8	6	5
ANNMAX flood peak constraint	769	1457	2991	2641
Flow duration curve constraint	727	1263	2892	520

Table 2 provides a comparison of the parameter ranges of the initial and behavioural TOPMODEL parameter sets for the Wye catchment. It is apparent that the majority of the behavioural parameter ranges (those of *SRMAX*, T_0 and *STDT*) are consistent with those of the original sampling ranges. The exception occurs with respect to the *m* parameter, which is important in controlling the rate of contributing area expansion and the hydrograph recession response. Similar results have been obtained for the other four catchments, and these findings support Beven's (1993) concept of equifinality.

Table 2. Initial and behavioural TOPMODEL parameter ranges for the Wye catchment

Parameter	Initial range	Behavioural range
m (recession)	0.0075 : 0.0450 m	0.0089 : 0.0152 m
SRMAX (maximum root zone storage)	0.0010 : 0.2000 m	0.0016 : 0.1995 m
T_0 (transmissivity)	1.0000 : 8.0000 log	1.4403 : 7.9991 log
STDT (standard deviation from transmissivity)	1.0000 : 10.0000 log	1.0048 : 9.1579 log

Figure 1 provides two examples of the likelihood weighted uncertainty bounds obtained for flood frequency (based on 1000 year continuous hourly simulations) for the Wye catchment. The light solid lines are the median and 90% uncertainty bounds for current climatic conditions. Twenty-one years of observed hourly annual maxima data (together with their corresponding GEV fit) are also displayed. It can be seen that the uncertainty bounds adequately bracket the observed data. The methodology is therefore an attractive alternative to traditional statistical analysis of flood frequency. In addition, the methodology can, potentially, reflect the changing nature of hydrological processes with more extreme events.

The dotted and dashed lines are the median and 90% uncertainty bounds derived from a climate change scenario for the 30 year timeslice of the 2080s. In this particular example, the changes to monthly rainfall and annual potential evapotranspiration (estimated from the HadCM2 UKCIP98 climate change scenarios

for the grid box containing Wales, UK (Hulme and Jenkins, 1998)) were directly applied in a uniform manner to the simulated hourly rainfall and potential evapotranspiration. The results are directly comparable with those of the current condition scenario. It is interesting to note that, although flood frequency risk does increase in the 2080s scenario, there is a substantial overlap of uncertainty between the current conditions and 2080s. This result has also been obtained for many other climate change scenarios for flood frequency for the Wye catchment (Cameron *et al*., 2000b). This highlights the importance of incorporating uncertainty analysis in estimating the impacts of climate change upon flood frequency.

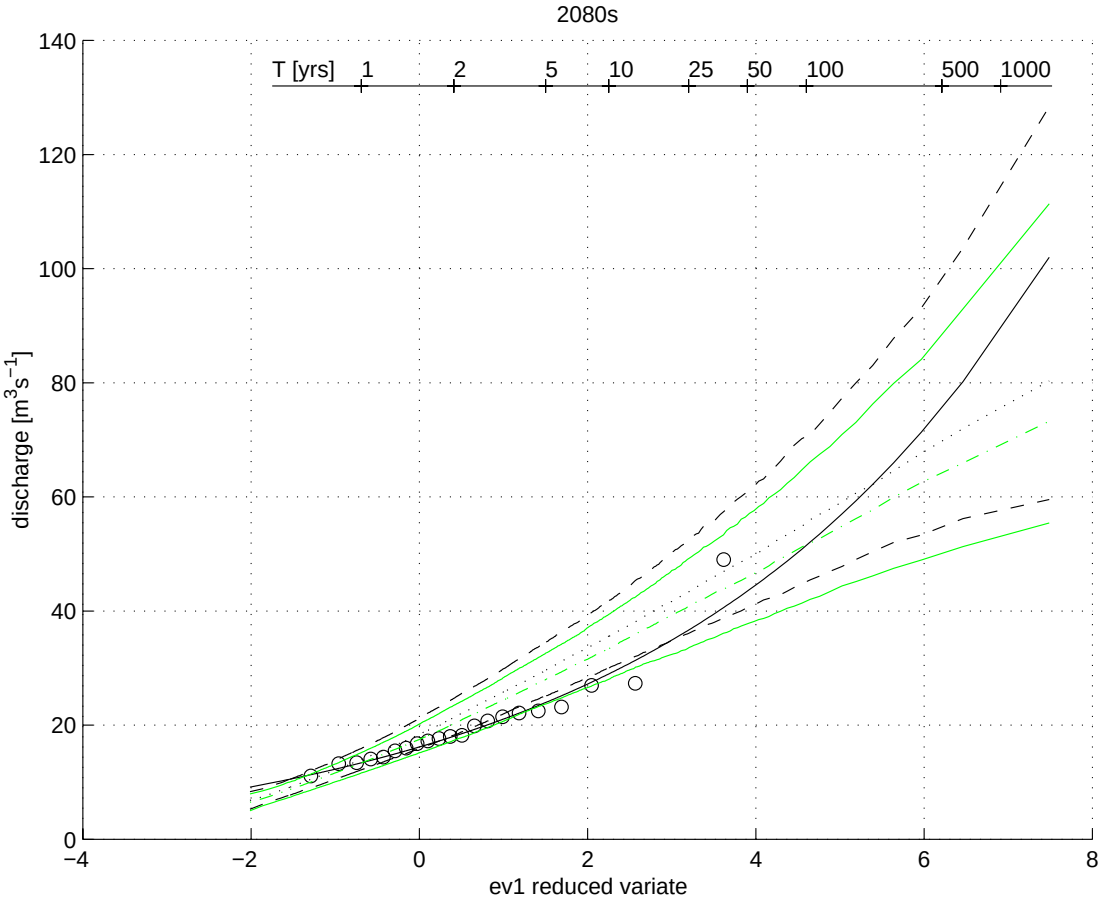


Figure 1. 90% uncertainty bounds derived from annual maximum peaks of 1000 behavioural TOPMODEL/rainfall parameter sets with 1000 year simulation length. Circles – observed hourly peaks; dark solid line – observed series with fitted GEV (maximum likelihood); dashed lines – 90% uncertainty bounds obtained from climate change scenario for 2080s; dotted line – median simulation obtained from climate change scenario for 2080s; light solid lines – 90% uncertainty bounds obtained for current conditions; light dash-dot line – median simulation obtained for current conditions.

CONCLUSIONS

A continuous simulation methodology for flood frequency analysis which explicitly accounts for uncertainty has been described. The methodology consists of a rainfall-runoff model, a stochastic rainfall model, and an uncertainty estimation procedure (the Generalised Likelihood Uncertainty Estimation, GLUE, technique of Beven and Binley, 1992). The methodology compares favourably with the traditional statistical

approach to flood frequency. It can also be used to investigate the impacts of climate change upon flood frequency. Given an appropriate choice of rainfall-runoff and rainfall models the approach can easily be adapted for use in many geographical locations.

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Generation of spatially consistent rainfall data

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P.J. Northrop, D.R. Cox and D. Koutsoyiannis

INTRODUCTION

Rainfall is the primary input to river flows. For continuous flow simulation therefore, one possible approach is to develop a continuous simulation model for rainfall, and to route its output through runoff models to obtain simulated flow sequences. Here, we review some recent developments in stochastic rainfall modelling. The aim is to provide rainfall simulations that are appropriate for hydrological application.

The next section deals with the development of models that can be calibrated using radar rainfall data. Such data represent an important source of information regarding the fine-scale structure of spatial rainfall fields. However, recognising their current limitations with respect to accuracy and record length, we subsequently describe some alternative methods that can be used when radar data are scarce, unreliable or absent. A more detailed account of the work is given by Wheater *et al*. (2000).

SPATIAL-TEMPORAL MODELLING USING RADAR DATA

There are several different approaches to the modelling and simulation of precipitation. In this section we focus upon a category of stochastic models, which aim to provide a simplified conceptual representation of the rainfall process in continuous space and time. This is done using probabilistic assumptions that enable rainfall structures to be represented using a few physically interpretable parameters. Such an approach is particularly suited to continuous simulation in the context of flow generation: the models' representation of rainfall in continuous space and time gives complete flexibility of application, and they are cheap to simulate.

Our approach is a spatial analogue of models that have been used to represent the temporal process of rainfall at a single site (e.g. Rodriguez-Iturbe *et al*., 1987; 1988). The models use a hierarchical representation of rainfall, in which the smallest element is a *rain cell* – a localised area of relatively intense rainfall. It is known that cells tend to cluster within larger-scale structures that we call *storms*. These storms themselves cluster, to form *rain events.* At the spatial scale of UK catchments, these correspond to weather systems. We treat the interior of a rain event as a stationary stochastic process. However, there are many different types of event, and the parameters of this process will typically vary between event types. We therefore construct, for each month of the year, a library of model parameters to reflect the prevailing types of weather system during that month.

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Modelling event interiors

In our model, a rain event consists of a collection of storms. Storm *centres* originate in a Poisson process in space (in practice, this means some large region containing the area of interest) and time, and they move with a constant velocity during their lifetimes. Each storm consists of a cluster of elliptically shaped rain cells, scattered about the storm centre in both space and time and moving with the same velocity as the storm centre. Each cell deposits rain at a constant rate (or intensity) over its entire area, and throughout its lifetime. The numbers of cells, their intensities, shapes and sizes, their temporal durations and their displacements about the storm centre, are random variables. Various independence assumptions are made, but are not discussed here. The spatial structure of a storm is shown schematically in Figure 1.

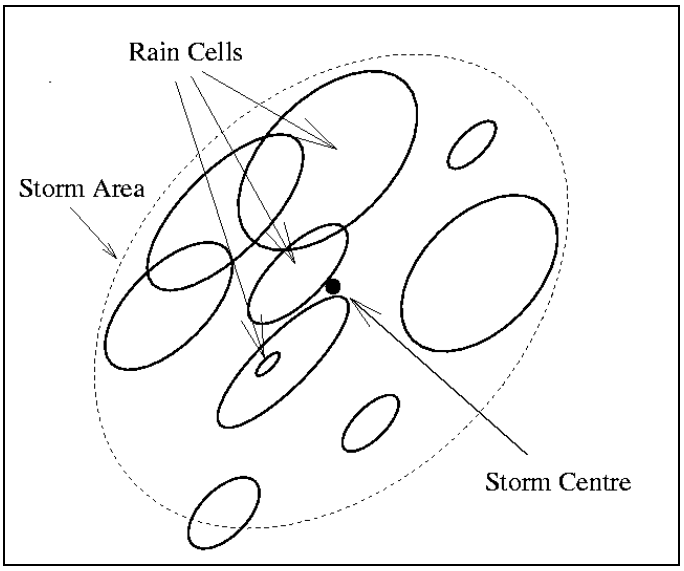


Figure 1. Schematic diagram showing the spatial structure of storms within the spatial-temporal model for event interiors

In the model used here, the spatial distribution of cells about a storm centre is bivariate normal. We therefore refer to the model as the *Gaussian Displacements Spatial-Temporal Model* (GDSTM). By varying the parameters of the displacement distribution, storms with different characteristics can be produced. For example, we can obtain 'scattered showers' if the variance of this distribution is large relative to the mean cell size. Rain bands result from a distribution with elongated elliptical contours. For simplicity, the shapes of cells within a storm are taken to be the same as the elliptical contours of the cell displacement distribution. A full description of the model requires distributions to be specified for all of the random components. Details may be found in Wheater *et al*. (2000), and in Northrop (1998).

The GDSTM has 11 parameters. These are physically interpretable quantities such as the expected cell duration, mean cell intensity and expected number of cells per storm. They can be estimated from a sequence of radar images by selecting some properties of the data, and finding parameter values that reproduce them as closely as possible (usually in a weighted least-squares sense). These properties might include the mean and variance of rainfall intensities, spatial and temporal autocorrelations at different scales, and the probability that an arbitrary region experiences no rainfall during a specified period. Their values, under the model, can be derived from its continuous space-time representation – see Northrop (1998) for details. Numerical methods are required to estimate the parameters from a given set of fitting properties. To ensure the stability of such methods, it is helpful to estimate groups of parameters separately where possible. Section 2.8 of Wheater *et al*.

(2000) contains some discussion of the issues involved. In general, parameter estimates for the GDSTM agree closely with previous studies, with respect to such quantities as cell and storm size and duration.

Validation of the GDSTM can be carried out by comparing observed and theoretical properties not used in the fitting procedure. For use as input to a continuous flow simulation, it may be important to reproduce relatively complex properties of rainfall fields, such as the extent and duration of localised areas of heavy rainfall (which can be quantified using suitable summaries of a rainfall sequence in which all rainfall intensities below some high threshold are set to zero). Such properties cannot be determined analytically, but can be explored by simulating the fitted model.

To illustrate the appearance of realisations from this model, Figure 2 shows an hourly sequence of radar images from the Wardon Hill radar station in south-west England,

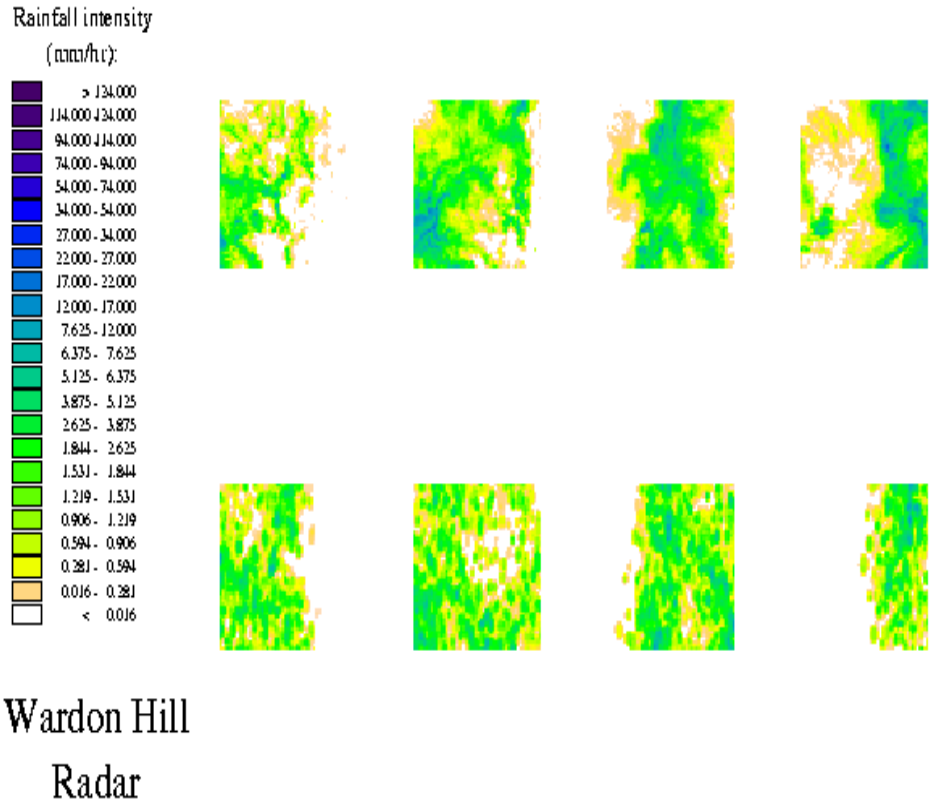


Figure 2. Visual assessment of the GDSTM fitted to the rain event of 6 February 1994, 13:00-14:00. The temporal separation of the images is 1 hour. Top: Observed data. Bottom: Simulation

together with an hourly sequence of images simulated from the GDSTM fitted to this event. The images are at $2\times2\text{km}^2$ spatial resolution, and cover an area of $104\times104\text{km}^2$. The model parameters were estimated from a 1-hour sequence of 5-minute radar images from the middle of this event. Figure 2 includes time periods outside that used for model fitting, to illustrate the movement of the rain event over the area (see the following Section). The observed and simulated sequences exhibit broadly similar features, although the simulations have a more obvious cellular structure than the data. Quantitative assessments

of fit, described in Wheater *et al*., (2000), indicate that the model reproduces many hydrologically significant features of this event.

The GDSTM has been fitted to data from a total of 207 rain events, recorded at Wardon Hill between September 1993 and March 1997. The events have been split by month, to allow for seasonal variations. We therefore have 12 monthly libraries of parameter estimates that can be used to generate different types of events as appropriate in each month of the year.

Modelling event sequences

The GDSTM is stochastically stationary in both space and time, and hence is only appropriate for modelling the interiors of rain events. To obtain realistic continuous simulations of rainfall sequences, we need to model the process by which events enter and leave a catchment. Wheater *et al*. (2000) give the motivation, and supporting data analyses, for the scheme described here. We define two types of event: *Type 1* events are those that extend over the entire area of interest for part of their lifetime, and *Type 2* events are those that may produce a significant amount of rainfall, but do not cover the entire area. Analysis of Wardon Hill radar data indicates that, even at spatial scales as large as 10,000km^2, Type 1 events are responsible for a large proportion of the total rainfall. We model the stochastic process of events and dry periods using a semi-Markov process with 3 states ('dry' periods, *Type 1* events and *Type 2* events). The durations of sojourns in each state are independent random variables, modelled using Weibull distributions. For simulation purposes, events of any duration can be generated by simulating the GDSTM within a quadrilateral, of appropriate size, that moves across the catchment. The random geometries of these quadrilaterals are determined by analysing the edges of observed rain events. For the simulation in Figure 2, the quadrilateral is an approximately rectangular region, aligned in a North-South direction and moving from west to east: the leading (eastern) and trailing (western) edges appear in the first and final images respectively.

Seasonality is an important feature of rainfall sequences. It can be incorporated into the process of event arrivals, by fitting separate Weibull sojourn distributions for each month of the year. To reproduce seasonal variations in event types, in any month we generate events using GDSTM parameter sets sampled randomly from the corresponding library of parameter estimates. The overall algorithm for continuous simulation is as follows: (i) generate the time until the next event, by sampling from the current month's Weibull distribution for dry period lengths (ii) determine the event type, according to the appropriate frequencies for the month (iii) determine the event duration by sampling from the appropriate duration distribution (iv) select a GDSTM parameter set from the current month's library (v) simulate the GDSTM within a moving quadrilateral, to give the correct duration (vi) return to step (i).

The performance of this scheme has been investigated by comparing properties of simulations with those of observed radar sequences. Comparisons have been done for data at time scales of 15 minutes, 1 hour and 1 day, and at spatial scales ranging from 2×2km^2 up to 104×104km^2. Various properties have been investigated, ranging from basic statistical measures (means, variances, autocorrelation functions and proportions of an area that are dry) to analyses of extreme behaviour. In general, the scheme reproduces

a wide range of features of observed rainfall fields. Again, details may be found in Wheater *et al*. (2000).

ALTERNATIVES, FOR APPLICATION IN THE SHORT-TERM

At present, the continuous simulation scheme described above has two drawbacks. The first is its reliance on radar data, which are limited both in terms of availability and quality. Realistically, it may be 5-10 years before adequate data are available to support the scheme operationally in the UK. A simplified version of the model, which may be calibrated using sub-daily data from a raingauge network, is described in Section 5.1 of Wheater *et al*. (2000). However, it has been found that even dense gauge networks do not provide sufficient spatial information to identify features on the scale of rain cells. Moreover, the availability of sub-daily gauge data is also limited in the UK at present.

The second drawback is that, except for seasonally varying parameters, the scheme is stationary in both space and time. It may therefore be inappropriate, in its present form, for use in regions with a lot of topographic variability, or for the generation of long rainfall sequences when long-term changes in climate are suspected. It is anticipated that this may be overcome by further model development. However, at present it may be appropriate to consider alternative methods, which are suitable for application in the short term.

Generalized Linear Models

Although radar and sub-daily raingauge data are relatively scarce in the UK at present, long *daily* records are more abundant. These can be used to study non-stationarities in rainfall sequences, and to develop daily rainfall simulation models. In principle, the output from such models can be downscaled to any desired resolution using some appropriate method. Hourly data may be adequate for most flow simulation purposes.

Many techniques are available for generating daily rainfall sequences. Here, one of the primary concerns is the incorporation of non-stationarities. A powerful and flexible technique, which allows non-stationarities to be quantified and incorporated, is that of Generalized Linear Models (GLMs). These are standard in the statistical literature, and were introduced into the study of daily rainfall by Coe and Stern (1982). The basic idea, which is an extension of linear regression, is to use the values of various predictors to forecast a probability distribution for the amount of rainfall at a site on a given day. In fact, this distribution is specified in two parts – we model the probability of rainfall occurrence separately from the amount of rain if non-zero. Previous days' rainfall amounts can be used as predictors, to account for autocorrelation. Other predictors might include quantities representing regional variation (such as site altitude), seasonal variability, long-term trends and 'external' climatological factors such as the North Atlantic Oscillation (NAO). Models can be specified in such a way that the effect of a predictor depends on the values of others – for example, it is known that the NAO affects European climate predominantly in the winter months. Models can be fitted, and compared, using Maximum Likelihood; they can be tested using a variety of simple but informative checks. By defining suitable dependence structures between sites, it is possible to build a multivariate GLM that allows simulation of non-stationary daily rainfall sequences over a network of sites.

The GLM methodology has been applied to many rainfall datasets from the UK and elsewhere. It has been found to be extremely useful for interpreting historical records,

particularly when we want to study changes in the climate of an area. Moreover, providing models have been specified carefully, GLM simulations can reproduce a variety of features (including extreme behaviour) of observed rainfall sequences. Chapter 4 of Wheater *et al*. (2000) contains technical details and a selection of results. Other applications include: a flooding study in Ireland, where GLM outputs were downscaled to hourly resolution for input to continuous flow simulations (OPW 1998); and an investigation into drought in the Yangtze River in China (Yang, 2001).

Spatial-temporal rainfall disaggregation

The GDSTM can provide stationary sequences in continuous space and time, whereas the GLM can provide non-stationary sequences at a daily timescale. It is natural to try and combine these two techniques, so as to obtain the advantages of both. Various methods of doing this have been explored. Perhaps the most promising to date uses a method due to Koutsoyiannis (2001); this is described in Section 5.3 of Wheater *et al*. (2000). The method is appropriate for use when one or more sub-daily time series are available, along with several daily time series at a network of sites. The sub-daily time series may be either observations, or simulated data from a single-site version of the GDSTM, or data produced by disaggregating a simulated daily series. The daily series may be observational, or may have been generated by simulating a GLM. The basic idea is to generate hourly rainfall sequences at the sites with daily data, that respect both the fixed daily totals and the spatial structure of sub-daily rainfall fields. This spatial structure can be deduced from a simplified version of the GDSTM, fitted to the available data. The procedure produces hourly sequences with the correct means, variances, coefficients of skewness and proportions of dry intervals. Promising preliminary results are reported in Wheater *et al*. (2000).

SUMMARY

Continuous simulation of rainfall fields, incorporating fine-scale spatial and temporal structure, can only be achieved if supported by adequate radar data. Models such as the GDSTM appear to offer the potential for generating realistic scenarios, once adequate data become available (although further work is required to address issues of non-stationarity). In the meantime, other methods using downscaled daily sequences may be more feasible. Such daily sequences may be generated using GLMs. Although these methods cannot describe the full spatial structure of rainfall fields (they are essentially multi-site models), they are extremely powerful and have a lot of potential for hydrological application. An ongoing research project, sponsored by DEFRA, is carrying out a detailed investigation of some of the methods reported here, with a view towards making recommendations for practitioners. A report, using case studies to illustrate the techniques, is at an advanced stage of preparation.

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Continuous simulation as a basis for national flood frequency estimation

Robert Lamb and Ann Calver

INTRODUCTION

The paper presents a generalised approach to estimating river flood frequencies based on rainfall-runoff modelling. The modelling is on a continuous time basis, rather than just for flood events, and the estimation procedure is generalised in space to be applied at ungauged sites as well as those with calibration data.

The method is generic in principle. It is being developed for national application in Great Britain, using a sample of calibration sites. It may be viewed as the 'next generation' flood frequency estimation methodology following the Flood Studies Report (Natural Environment Research Council, 1975) and the Flood Estimation Handbook (Institute of Hydrology, 1999) which provide methods based on statistical analyses and event-based hydrograph approaches. Whilst continuous simulation can be applied at a detailed level for specific catchments, this paper focuses on spatially-generalised (or 'regionalised') application, whereby flood frequencies are modelled for any site, gauged or ungauged.

MODELS AND CALIBRATION

In principle, any appropriate rainfall-runoff model can be used to simulate flood responses in a particular catchment. For a national system, to be used for ungauged as well as gauged sites, certain considerations become important in choice of model. For the runoff models that are ultimately to be used for sites without flow data, generality of model structure is clearly important, but this must be balanced with parametric efficiency to mitigate problems of parameter uncertainty. This becomes especially significant when attempting to relate model parameters to catchment properties data because functional dependence between parameters can lead to great uncertainty in the derivation of the required empirical relationships. In the work reported here, two runoff models developed at the Institute of Hydrology were used, the Probability Distributed Model (PDM) of Moore (1985, 1993) and the Time-Area Topographic Extension (TATE) model of Calver (1993, 1996). These both have a variety of formulations and have been used in flood frequency modelling contexts with three to seven parameters. Both are conceptual stores-and-transfers hydrological models with an interpretation in physical process and utilise distribution functions to summarise spatial distributions of catchment runoff production.

Rainfall and river flow data are needed in order to establish model parameters for the sample sites. These data should be of as good hydrometric quality as possible, and as complete and homogenous as possible, particularly in the case of the rainfall

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data. It is a strong advantage for the sample of catchments to include examples of the whole physiographic and climatic range for which flood frequency estimates will ultimately be sought from the spatially generalised method. This work has used hourly data in order to capture flood detail without excessive data storage. The techniques used for quality control and data infilling have been described by Lamb and Gannon (1996).

The choice of calibration method for model parameters at each sample catchment is related in part to the way subsequent relationships to catchment properties are sought (see below). Problems of calibration uncertainty in hydrological modelling are well-known; pragmatic solutions are, however, required in order to offer flood frequency estimates in a generalised framework. The approach of establishing a single set of model parameter values for a particular catchment can be valuable, provided care is taken in the calibration process. In general the experience in this work is that when automatic 'optimisation' techniques are used to determine parameter values considerable benefit is gained from also reviewing the results using hydrological judgement.

Objective functions which have been found to be helpful in quantitative comparisons include those related specifically to flood peak magnitudes (Lamb, 1999) as well as to overall goodness of fit of modelled and observed flow time series. The flood frequency curve can be used directly as the primary measure of assessing goodness-of-fit, but confidence in the modelling basis is enhanced by realistic overall characterisation of the time series, including hydrograph shapes and flow duration curves. It is an assumption in model calibration (as in most statistical frequency analysis) that catchment data are effectively stationary with respect, for example, to river structures and catchment management practices. It is not necessary for the rainfall regime to be stationary if the results of modelling are expressed as time series, but for expression as, for example, a fitted flood frequency distribution, climatic stationarity is assumed over the period to which the curve relates.

Information on model parameter values can also be established by sampling according to physically-realistic distributions of possible values for each parameter and thereby deriving distributions of chosen measures of goodness-of-fit in multi-dimensional parameter space. Model run numbers in the order of 10000 have been used to derive these distributions for three- to five-parameter models.

Figure 1 demonstrates the level of calibration achieved by parameter sparse models (in this case the PDM) on two catchments of contrasting flood response. Flood frequency curves have been derived from the modelled time series using standard partial duration series analyses (e.g. Naden, 1992) here using Poisson distributions for arrival times of peaks and generalised Pareto distributions for magnitudes.

GENERALISATION TO UNGAUGED SITES

The catchment property data which allow generalisation of the method should ideally be easily obtainable, and offer physically plausible explanation of variance in calibrated model parameters. In practice, it may have to be accepted that some catchment properties function in a surrogate rather than in a direct way and, in doing so, may combine the effects of some less succinctly expressed properties. The major categories of catchment properties used were geological and soil material properties, catchment geometry and river network geometry indices, land-use and climate

properties and standard hydrological indices (that can be derived without site flow data). Flood Estimation Handbook catchment descriptors are included.

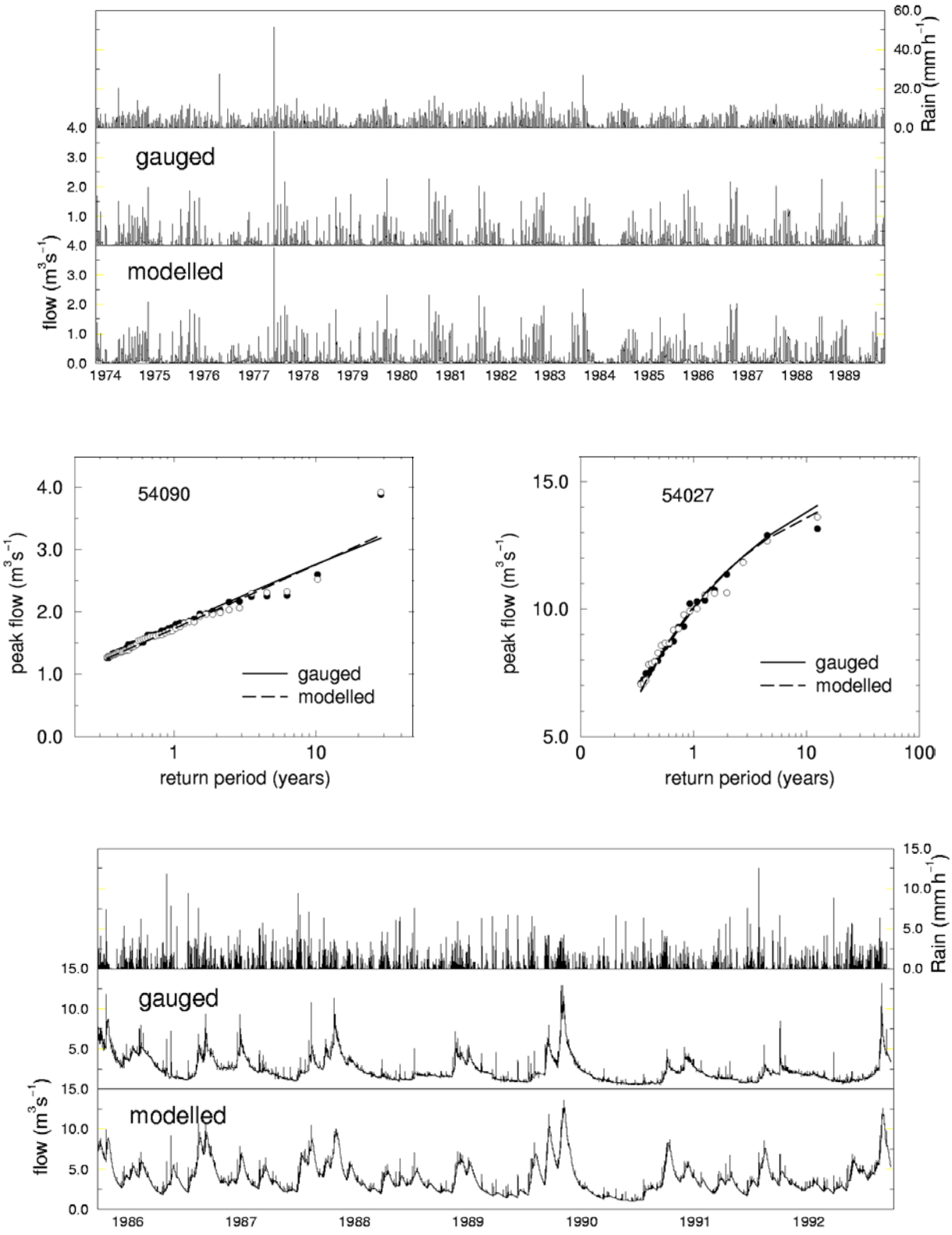


Figure 1. Calibration simulation results for two gauged sites of contrasting responses; (top) 54090 the Tanllwyth, mid Wales and (bottom) 54027, the Frome at Ebley Mill. Both simulations used the same simple, general conceptual model structure.

Fitting relationships between model parameters and catchment properties is a difficult problem. Both simple and more complex approaches have been tested. Conventional univariate multiple regression has provided encouraging results in a number of cases, whilst a more sophisticated 'sequential' regression approach has been developed to accommodate to some degree the dependence between model parameters.

Pilot study results of applying univariate multiple regression to relate single 'best' estimates of model parameters to catchment properties were reported by Calver *et al.* (1999). This approach necessarily assumes that model parameters are independent, which is not entirely the case. In the univariate approach, calibrated values of model parameters at gauged sites are essentially treated as if they were 'independent observations' of the parameters. Multivariate multiple regression, recognising parameter inter-dependencies, requires assumptions to be made on the form of these dependencies and is difficult to apply in practice to this situation.

A new 'sequential generalisation' approach aims to account for parameter covariance with few restrictive assumptions. In this approach the model parameter predictor equations were derived sequentially, accounting for effects that generalisation of earlier parameters have on later parameters. The approach is outlined in Figure 2, and more detailed descriptions can be found in Crewett *et al.* (1999, 2000) and Lamb *et al.* (2000a,b). In essence the approach seeks to address, to a practical degree, the difficulties of model equifinality, and to handle the assumptions of regression techniques in a way that is practical for conceptual hydrological modelling.

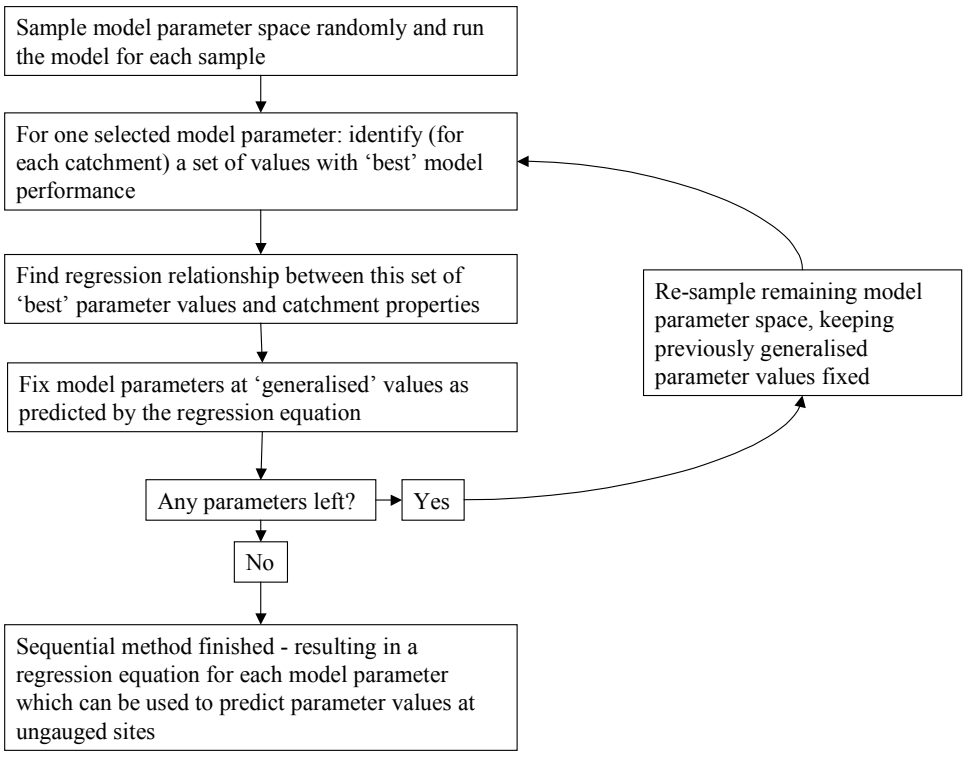
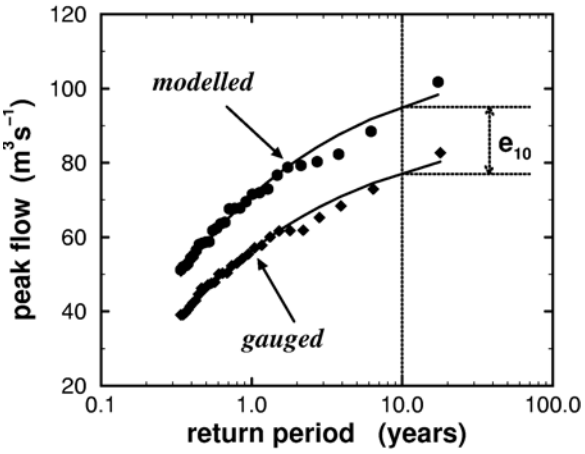


Figure 2. Outline flowchart for sequential calibration and generalisation

Once model parameters can be predicted from spatial catchment properties data, the model(s) can be run for any site, given driving series of rainfall values. River flow time series can thus be derived, together with flood frequency and flow duration curves.

Figure 3 summarises the current level of performance of the method when applied to sites which have been treated as though ungauged, but with the real flow data retained for test purposes only. Part A of the figure shows how the measures of performance were derived and part B gives the numerical values for recurrence intervals up to lengths appropriate to test in terms of the observation record length.

(a)



(b)

	Return period (years)	*1.0*	*2.0*	*2.33*	*5.0*	*10.0*	*20.0*
Model: TATE							
Pilot study*	Mean % error	31	31	32	33	35	38
	SD % error	29	31	32	33	35	36
New results (TATE3)	Mean % error	25	24	24	25	28	31
	SD % error	23	22	22	23	26	33
Model: PDM							
Pilot study*	Mean % error	42	40	40	40	42	45
	SD % error	35	33	32	34	37	44
New results (sequential)	Mean % error	22	23	24	24	26	27
	SD % error	18	18	19	20	21	23

**Calver et al. (1999)*

Figure 3. (a) Schematic illustration of the 'residuals' used to assess flood quantile estimates. Individual errors e_T are subsequently scaled, so as to be expressed as a percentage of the gauged peak flow estimate for return period T (= 10 years in the example). (b) Averages and standard deviations of these errors across all test cathments.

Data are given for the first pilot system (as in Calver *et al.*, 1999) and for subsequent improvements. These improvements (Lamb *et al.*, 2000a,b) result from the reduction in the number of model parameters (in the case of the TATE) and in the adoption of the sequential method (in the case of the PDM). Subsequent refinements can incorporate the advantages of both sources of improved performance.

UNCERTAINTY ESTIMATES FOR THE GENERALISED MODELLING

Estimates of uncertainty can be derived to accompany generalised as well as site-specific flood frequency estimates. Model parameters are expressed as statistical distributions and Monte Carlo simulations then used to generate ranges of simulated river flows from which flood peaks can be extracted. For the generalised case, first estimates of parameter uncertainty were obtained by sampling from the theoretical distributions about the regression estimates of the model parameters.

Lamb *et al.* (2000c) found that degrees of uncertainty varied between sites and were often no greater in the case of spatially generalised estimates than for site-specific estimates fitted to gauged flows. Three examples are shown in Figure 4, and compared with confidence intervals based on the likelihood for the single site, fixed-threshold generalised Pareto distribution fitted to each gauged partial duration series. Comparatively few sites indicated a 'failure' of the modelling or parameter generalisation.

EXTENSION OF METHOD FOR RARE FLOODS

For the estimation of high recurrence interval floods it is an advantage to model long time series of flows in order to cover rare floods without relying on extrapolation of the flood frequency curve, which can be very sensitive to exact fit to the lower-magnitude flow peaks. Very long observed rainfall records exist for some sites but, more usually, generated rainfall time series will be required. The field of spatio-temporal rainfall modelling appropriate for input to hydrological models is an area of on-going research. For pragmatic rainfall generator models, current practice tends to favour simple point (or catchment average) methods based on sampling from distributions parameterised on the basis of observed rainfall data. Such an approach employed here uses Goodsell and Lamb's (1999) modification of Blazkova and Beven's (1997) method.

In terms of the extension of the method in time it is possible, in addition, to perturb input rainfall time series to runoff models in accordance with expected climate changes. Examples of work on climate change effects on flood frequency in Britain using continuous simulation methods include Crooks *et al.* (1996), Reynard *et al.* (1999) and Calver *et al*. (1999). It should be noted that when climate change scenarios are based on output from general atmospheric circulation models, there is a range of possible interpretations of coarse-space, coarse-time data in disaggregation to catchment scales appropriate to flood modelling for design purposes. The implication of this is that informed caution needs to be exercised in the interpretation of results of flood frequencies under scenarios of future climates in order to reliably capitalise on opportunities afforded by continuous simulation methodology.

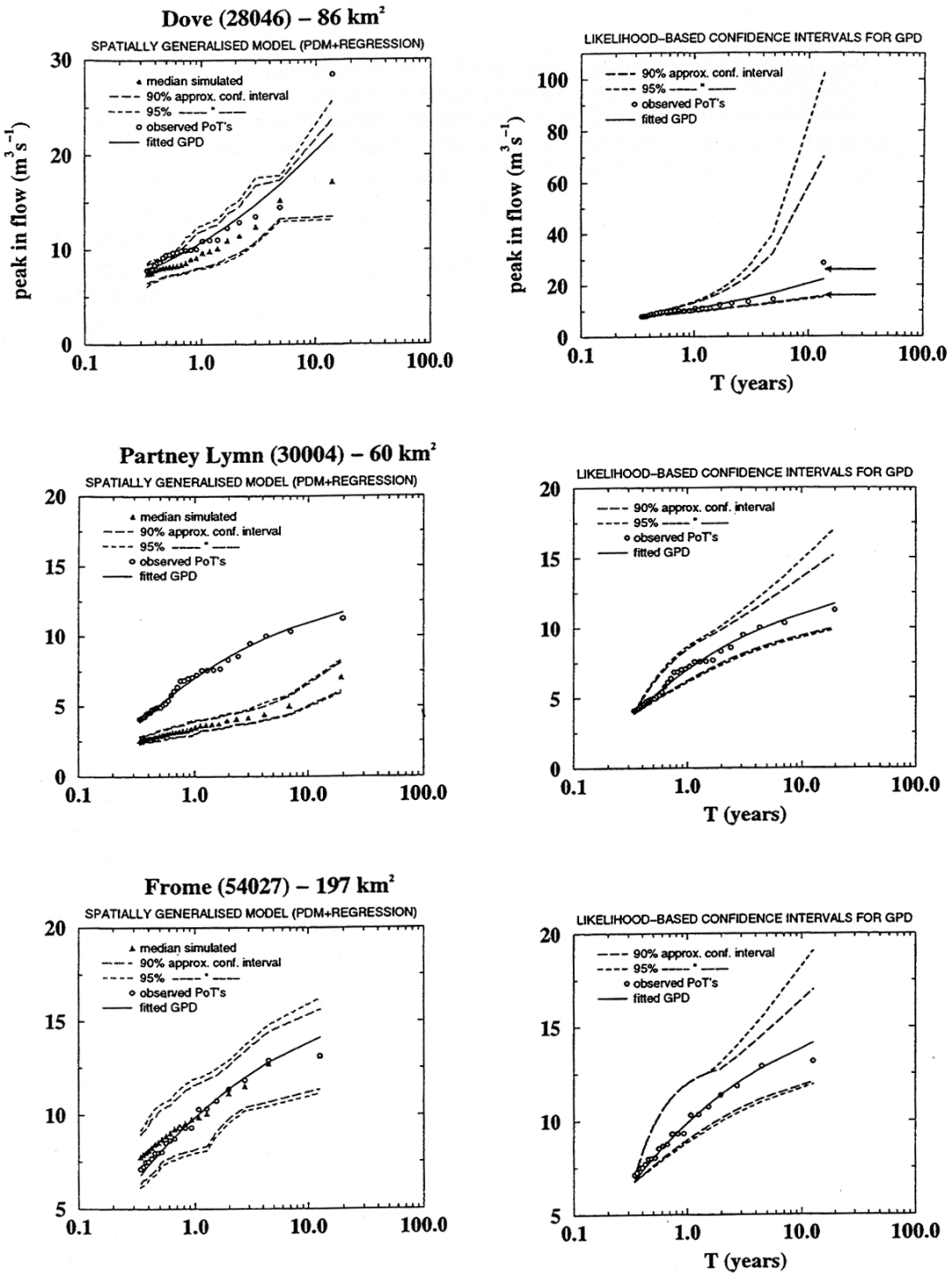


Figure 4. Uncertainty bounds for generalised flood frequency simulations (left hand panels), where 'unknown' gauged flows are shown for comparison, and single-site generalised Pareto distributions (right hand panels) fitted to the gauged flows. Three out of the 40 test catchments produced results suggesting 'failure' of model or generalisation procedure, as shown in the middle-left panel.

CONCLUDING REMARKS

The work discussed in this paper is a prototype system for flood frequency estimation in Britain using continuous simulation. It exists in the form of research computer codes, as opposed to, at this stage of development, an integrated user-friendly software product.

Confidence in the system to date is enhanced by the general level of success of model characterisation of the whole runoff response, including the flood response, across the conditions encountered in Britain. This includes catchments varying in area over three orders of magnitude and a mix of impermeable and permeable geologies. Further, when sites are treated as ungauged, and then the withheld observations are compared with spatially-generalised flood frequency results, recurrence intervals up to those which can be tested are within pragmatic acceptance levels. Advantages of the approach are:

- hydrograph volumes and durations are provided, as well as peak flows;
- there is no need to separate flood flows and base flows, since runoff processes are modelled as a whole;
- sequences of storms are readily handled in continuous simulation; and
- because antecedent wetness conditions are an integral part of the modelling, there is no requirement to define a relationship between rainfall recurrence intervals and those of design floods.

The production of time series of catchment runoff complements detailed river hydraulic modelling. It is, in addition, a relatively straightforward matter to incorporate aspects of climate change in that any rainfall time series can be applied to derive the runoff models to produce river flows.

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